Homework 2

Due: January 23, 2013 (January 21, 2013 is MLK day).

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7. 5.10

8. 5.13

9. 5.15

10. 5.17

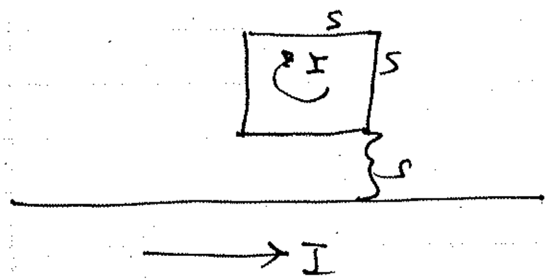
11. 5.19

12. 5.46

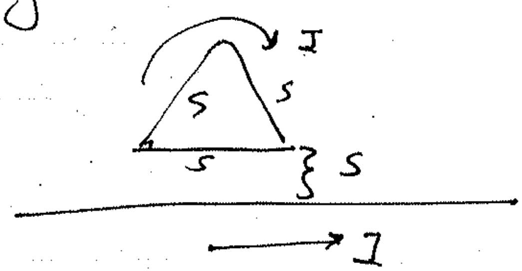
13. 5.49

Prob S.10

Find the force in the following situations:

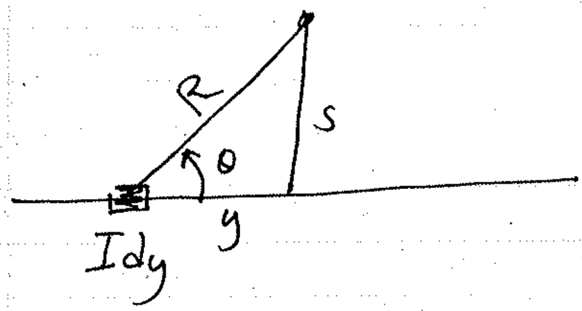


(a)



(b)

a) Find the field due to an infinite line charge



$$dB_\phi = \frac{\mu_0}{4\pi}\frac{I\,dy\sin\theta}{(y^2+s^2)}$$

$$B_\phi = \frac{\mu_0 I}{2\pi}\int_{-\infty}^{0}\frac{s\,dy}{(y^2+s^2)^{3/2}}$$

$$= \frac{\mu_0 I s}{2\pi}\left[\frac{+\cancel{4}y}{+\cancel{4}s^2\sqrt{s^2+y^2}}\right]_{-\infty}^{0}$$

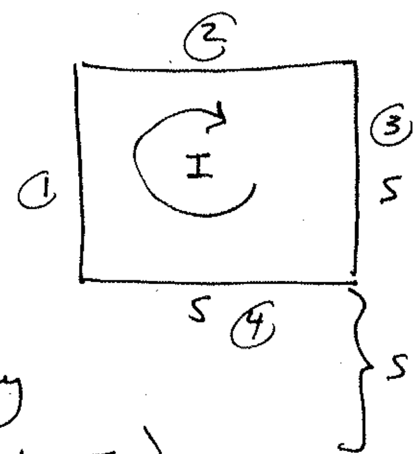
$$= \frac{\mu_0 I s}{2\pi}\left[\frac{+\infty}{s^2(\infty)}\right]$$

$$\boxed{\vec{B}_\phi = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

S.10.2

Calculate force

$$d\vec{F} = I d\vec{l} \times \vec{B}_\phi$$



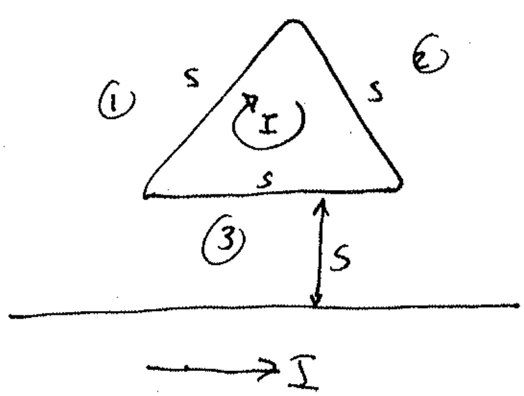
$$\vec{F}_{\parallel} = \vec{F}_1 + \vec{F}_2 = 0 \text{ by symmetry}$$

$$F_\perp = -\int_0^s I\,dl\left(\frac{\mu_0 I}{2\pi(2s)}\right) + \int_0^s I\,dl\left(\frac{\mu_0 I}{2\pi s}\right)$$

$$= \frac{\mu_0 I^2}{2\pi s}\left(-\frac{s}{2} + s\right)$$

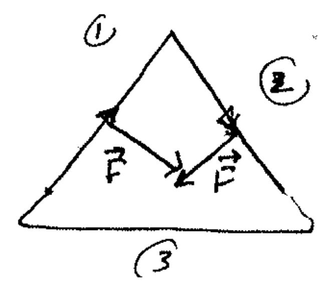
$$\boxed{F_\perp = \frac{\mu_0 I^2}{4\pi}}$$ upward

b)

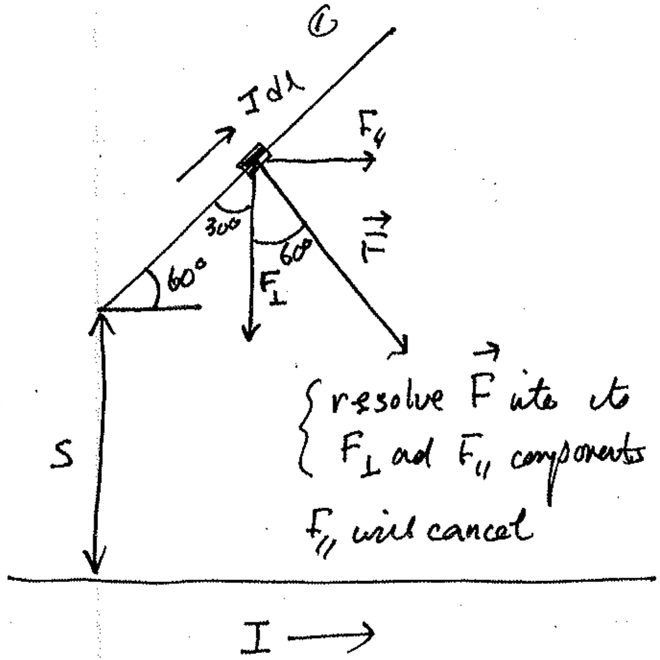


Side 3 $$\vec{F}_\perp = \int_0^s I\,dl\left(\frac{\mu_0 I}{2\pi s}\right) = \frac{\mu_0 I^2}{2\pi} \text{ (up)}$$

Side 1 & 3:



By symmetry, horizontal forces cancel.



{ resolve $\vec{F}$ into its
{ $F_\perp$ and $F_\parallel$ components

$F_\parallel$ will cancel

$$F_\perp = -\int_0^s I\,dl\left(\frac{\mu_0 I}{2\pi s}\right)\cos 60°$$

s depends on l

$$= -\frac{\mu_0 I^2}{2\pi}\cos 60° \int_0^s \frac{dl}{(s + l\cos 30°)}$$

$$= -\frac{\mu_0 I^2}{2\pi}\cos 60° \left[\frac{1}{\cos 30°}\ln\left(s + l\cos 30°\right)\right]_0^s$$

$$= -\frac{\mu_0 I^2}{2\pi}\ln\left[\frac{s(1+\cos 30°)}{s}\right]\frac{1}{\sqrt{3}}$$

for 1 side, the total force is

$$\vec{F}_\perp = +\frac{\mu_0 I^2}{2\pi} - \frac{\mu_0 I^2}{\sqrt{3}\pi}\ln\left[(1+\cos 30°)\right]$$

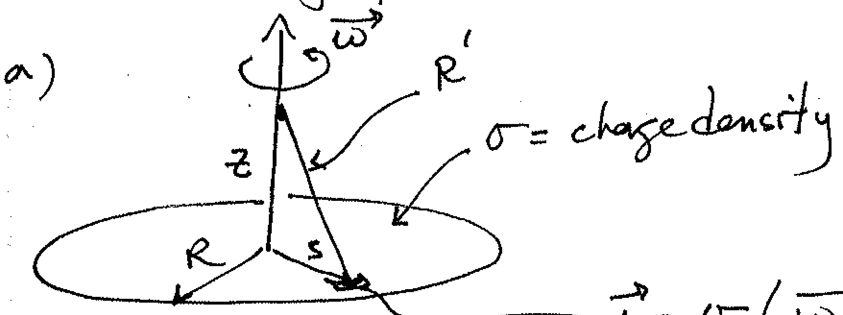
$$= \left(\frac{\mu_0 I^2}{2\pi}\right)\left[0.28\right] \uparrow \text{ up}$$

Prob 5.13

Find the field at a point $z > R$ on the axis of

(a) rotating disk
(b) rotating sphere.

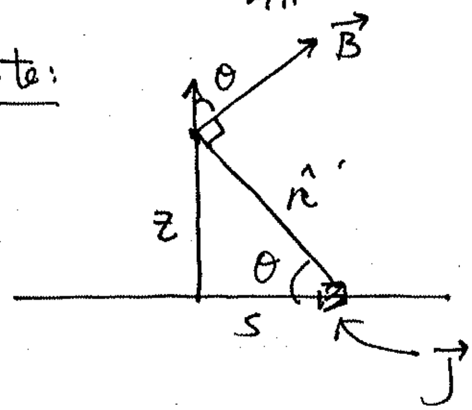
a)



$$\vec{J} = \sigma(\vec{\omega} \times \vec{r}) = \sigma \omega s \hat{\phi}$$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\left(\sigma \omega s \hat{\phi} \times \hat{R}'\right)\frac{1}{(s^2+z^2)}$$

note:



we only need to consider the vertical field.

$$\Rightarrow \vec{B}_z = \frac{\mu_0}{4\pi}\int \frac{\sigma \omega s^2}{(s^2+z^2)^{3/2}}\, ds$$

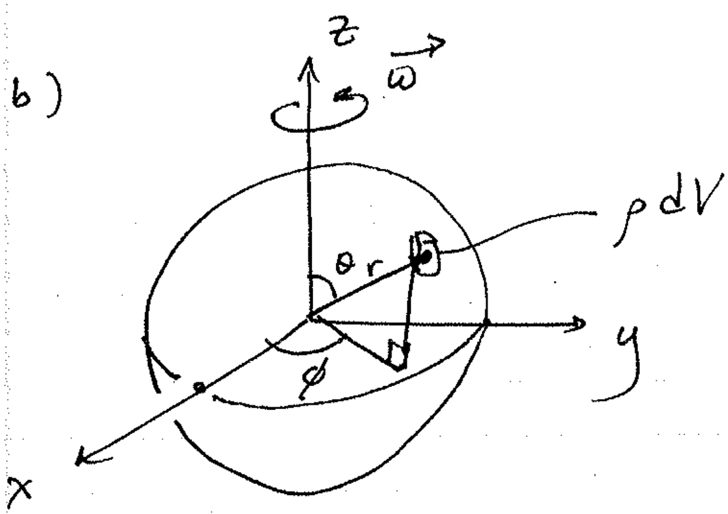
$$= \frac{\mu_0 \sigma \omega}{4\pi}\int_0^R \int_0^{2\pi} \frac{s^3\, ds\, d\phi}{(s^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 \sigma \omega}{2}\int_0^R \frac{s^3\, ds}{(s^2+z^2)^{3/2}}$$

Integrate to find

$$B_z = \frac{\mu_0 \sigma \omega}{2}\left[\frac{R^2+2z^2}{\sqrt{R^2+z^2}} - 2|z|\right]$$

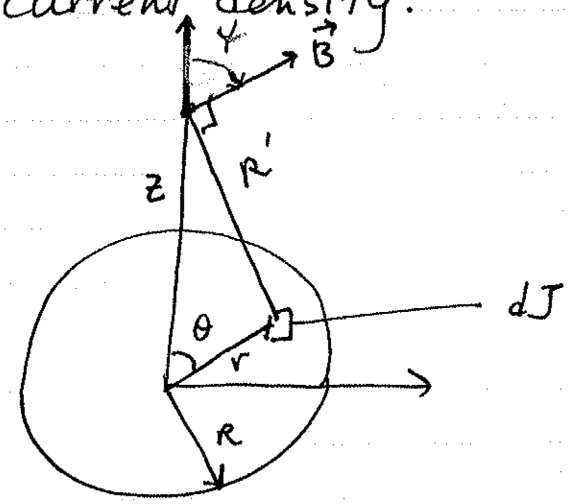
b)



(i) the current density is $d\vec{J} = \rho dV (\vec{\omega} \times \vec{r})$

$$= \rho dV \omega r \sin\theta \hat{\phi}$$

(ii) find the field on the rotation axis due to the current density.



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\rho dV \omega r \sin\theta (\hat{\phi} \times \hat{R}')}{(r^2 + z^2 - 2rz\cos\theta)}$$

again by symmetry, we need only keep the z-component of $\vec{B}$.

$\Rightarrow dB_z = dB \times \cos\psi$

$$\cos\psi = \frac{r\sin\theta}{\sqrt{r^2+z^2-2zr\cos\theta}}$$

(5.13.4)

$$B_z = \frac{\mu_0 \rho w}{4\pi} \int \frac{(r\sin\theta)^2}{(r^2+z^2-2zr\cos\theta)^{3/2}} r^2 dr \sin\theta \, d\theta \, d\phi$$

Integrate over ϕ

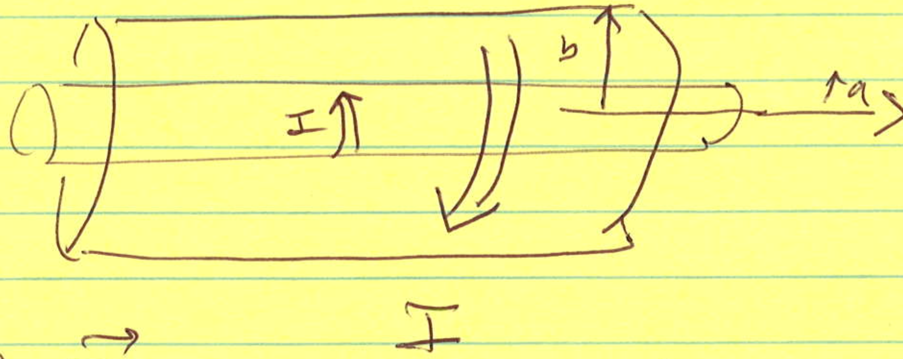
$$= \frac{\mu_0 \rho \omega}{2} \int \frac{r^4 dr \sin^3\theta \, d\theta \, d\phi}{(r^2+z^2-2zr\cos\theta)^{3/2}}$$

$$= \frac{\mu_0 \rho w}{2} \int_0^R \int_{-1}^1 \frac{r^4 dr \sin^2\theta \, d(\cos\theta)}{(r^2+z^2-2zr\cos\theta)^{3/2}}$$

$$= \frac{\mu_0 \rho w}{2} \int_0^R \int_{-1}^1 \frac{r^4 dr (1-\cos^2\theta) \, d(\cos\theta)}{(r^2+z^2-2zr\cos\theta)^{3/2}}$$

$$B_z = \frac{2\mu_0 \rho w}{15} \frac{R^5}{z^3}$$

Prob 5.15

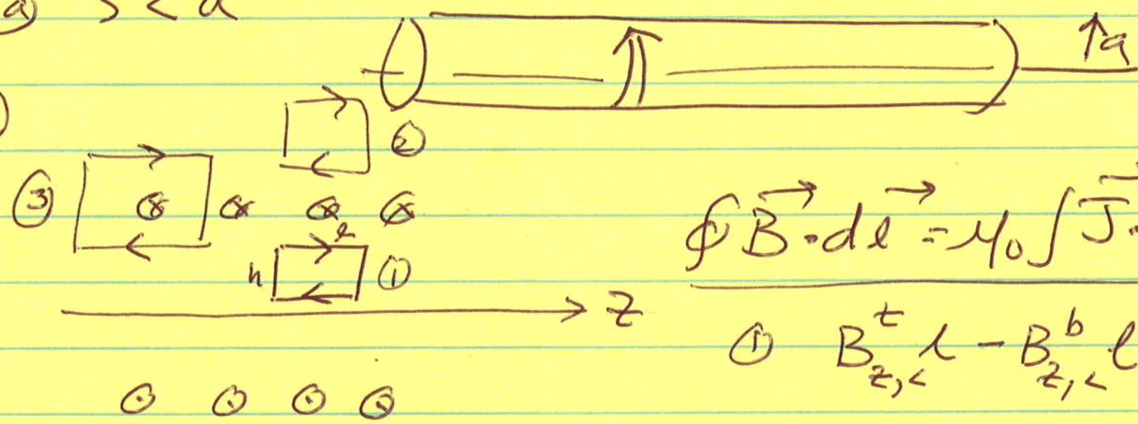


find $\vec{B}$ in (i) $s<a$, (ii) $a<s<b$, (iii) $b<s$

Sol^{n}

(a) $s<a$

(A)



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{s}$$

(1) $B^{t}_{z,<}\,l - B^{b}_{z,<}\,l = 0$

$\Rightarrow B_{z,<}$ is constant

(2) $B^{t}_{z,>}\,l - B^{b}_{z,>}\,l = 0$

and $B_{z,>} \to 0$ at ∞

$\Rightarrow B_{z,>} = 0$

(3) $\cancel{B^{t}_{z,>}}^{\,0}\,l - B^{b}_{z,<}\,l = \underset{n_1 l}{N}\mu_0 I$

$$\boxed{\vec{B}_{z,<} = -n_1\mu_0 I\hat{z}}$$

(B) Outer cylinder contributes, $\vec{B}^{outer}_{z,} = +n_2\mu_0 I\hat{z}$

$$\Rightarrow \boxed{\vec{B}_{Total} = \mu_0 I (n_2 - n_1)\hat{z} \quad , s<a}$$

(B) Between, $a<s<b$

$$\vec{B}_{Total} = \vec{B}_{inner} + \vec{B}_{outer}$$

$$= 0 + n_2 \mu_0 I \hat{z}$$

$$\boxed{\vec{B}_{Total} = \mu_0 n_2 I \hat{z} \quad , a<s<b}$$

(c) Outside, $b<s$

$$\vec{B}_{Total} = \vec{B}_{inner} + \vec{B}_{outer}$$

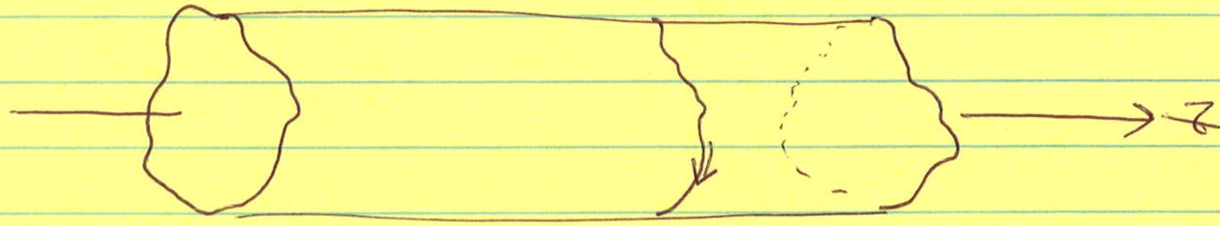
$$= 0 + 0$$

$$\boxed{\vec{B}_{Total} = 0 \quad , b<s}$$

Prob 5.17

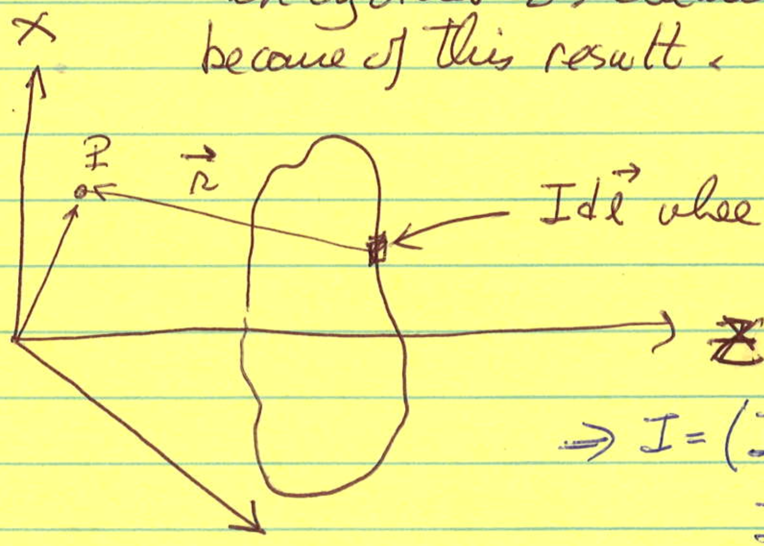
Show that the field, $\vec{B}$, of an infinite solenoid runs parallel to the axis, regardless of the cross-sectional shape of the coil, as long as the cross-sectional shape is constant.

Solution



(A) Because solenoid is long $\Rightarrow$ can't choose any z and find it different than any other z. Evaluate at $z = 0$ because of this result.

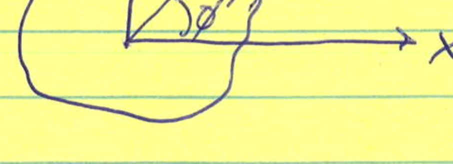
field point $\vec{P}$ is in xy plane $\Rightarrow \vec{P} = (x, y, 0)$



$I d\vec{l}$ where $\vec{I}$ has x, y components, but no z-component.

$\Rightarrow I = (I_s \cos\phi' - I_\phi \sin\phi',$
$I_s \sin\phi' + I_\phi \cos\phi',$
$0)$

project current loop onto xy plane



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{I}\,dl \times \hat{r}}{r^2}$$

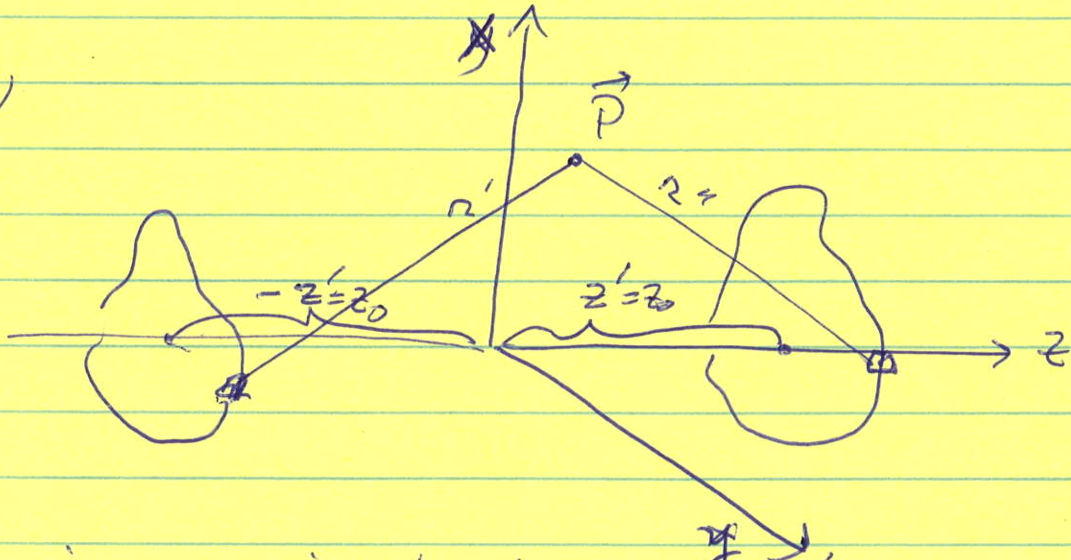
$$= \frac{\mu_0}{4\pi} \frac{\left(I_s\cos\phi' - I_\phi\sin\phi',\ I_s\sin\phi' + I_\phi\cos\phi',\ 0\right) \times \left(x - x',\ y - y',\ -z'\right) dl}{r^3}$$

$$= \frac{\mu_0\,dl}{4\pi r^3}\Big(-z'\left[I_s\sin\phi' + I_\phi\cos\phi'\right],\ z'\left[I_s\cos\phi' - I_\phi\sin\phi'\right],\ (y-y')\left(I_s\cos\phi' - I_\phi\sin\phi'\right)$$

$$- (x-x')\left(I_s\sin\phi' + I_\phi\cos\phi'\right)\Big)$$

now $(x', y', z') = (s'\cos\phi', s'\sin\phi', z')$

okay,

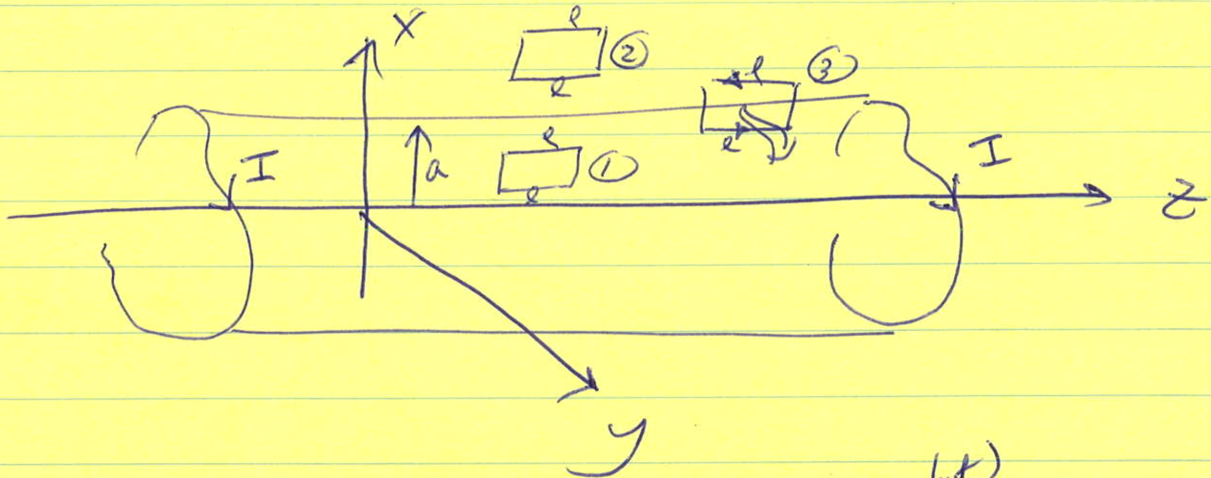


if we consider two loops at $z' = \pm z_0$ and look at field point $\vec{P}$ in xy plane $\Rightarrow |\vec{r}\,'| = |\vec{r}\,''|$, I_s, I_ϕ are ~~[illegible]~~ the same by symmetry and ϕ' is the integration variable $\Rightarrow$ x, y components in $d\vec{B}$ cancel leaving only z-component

$$d\vec{B} = \frac{\mu_0\,dl}{4\pi r^3}\left(0,\ 0,\ (y-y')\left[I_s\cos\phi' - I_\phi\sin\phi'\right] - (x-x')\left[I_s\sin\phi' + I_\phi\cos\phi'\right]\right)$$

$y' = s'\sin\phi'$ $\qquad\qquad$ $s'\cos\phi'$

~~also note that only terms $\propto I_\phi$ survive integration~~
~~over ϕ', but~~ let's use Ampere's law



② $\Rightarrow \oint \vec{B}_> \cdot d\vec{l} = 0 \rightarrow B_>$ continues, (constant) but $B_> \rightarrow 0$ at ∞

$\Rightarrow B_> = 0$

① $\Rightarrow \oint \vec{B}_< \cdot d\vec{l} = 0 \rightarrow B_<$ continues (constant)

③ $\Rightarrow \oint \vec{B} \cdot d\vec{l} = -0\,l + B_< l = \mu_0 \underbrace{n}_{\text{coils/length}} l I$

$$\rightarrow \boxed{\begin{aligned} \vec{B}_< &= \mu_0 n I \hat{z}, \quad < a \\ &= 0 \qquad\quad > a \end{aligned}}$$

N = total # of turns

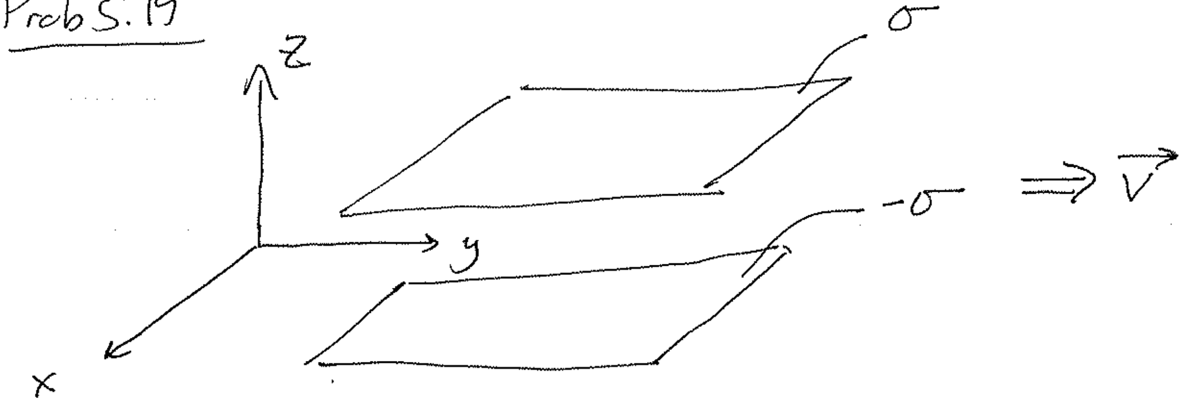
(B) the toroid has $\vec{B}_{Tor} = \dfrac{\mu_0 N I}{2\pi s}\hat{\phi}, \quad < a$

as $s \rightarrow \infty$, $\frac{a}{s} << 1$ and curvature becomes small

$\Rightarrow \dfrac{N}{2\pi(s+a)} \simeq \dfrac{N}{2\pi s}\left(1 - \dfrac{a}{s}\right) \simeq n \equiv$ turns per unit length

$\Rightarrow \boxed{\vec{B}_{Tor} \rightarrow \mu_0 n I \hat{\phi}, \quad < a}$

Prob 5.19



a) what is the magnetic field between the 2 charged plates?

Surface current density is $\vec{K} = \sigma \vec{v}$. The field due to one charged moving plate is

$$\vec{B} = \hat{x} \frac{1}{2} \mu J, \frac{z}{|z|}$$

where $\vec{v}$ is the $\hat{y}$ direction and the plate is placed at $z = 0$. So, place the plates at $z = \pm h$. In-between

(i) the plates the total field is

$$\boxed{\vec{B} = -\hat{x} \mu J}$$

(ii) above the plate at $z = +h$, and below the plate at $z = -h$,

$$\boxed{\vec{B} = 0}$$

b) what is the force per unit area on the top plate?

$$d\vec{F} = \sigma \vec{v} \times \vec{B}\, ds$$

$$= \sigma (0, v, 0) \times (-\tfrac{1}{2}\mu_0 J, 0, 0)\, ds$$

$$= +\tfrac{1}{2}\mu_0 J \sigma v\, \hat{z}\, ds$$

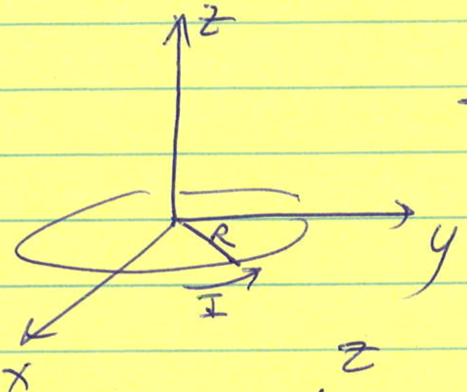
$$\boxed{\frac{d\vec{F}}{ds} = \frac{1}{2}\mu_0 J^2 \hat{z}}$$

c) at what v does this force (from b) balance the es attraction given by

$$\frac{d\vec{F}_{es}}{ds} = \sigma \vec{E} = -\sigma^2/2\epsilon_0\, \hat{z}$$

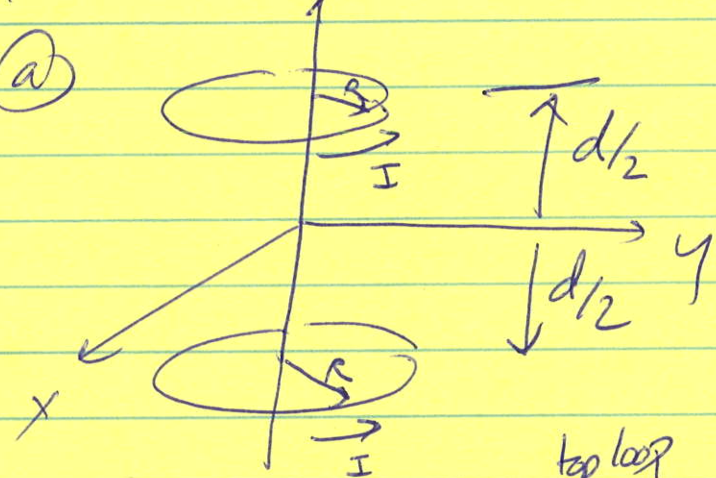
$$0 = \frac{d\vec{F}}{ds} + \frac{d\vec{F}_{es}}{ds} = \frac{1}{2}\mu_0 (\sigma v)^2 - \frac{1}{2\epsilon_0}\sigma^2 \Longrightarrow \boxed{v^2 = \frac{1}{\mu_0\epsilon_0} = c^2}$$

Prob 5.46



$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}} \hat{z} \text{ on axis}$$

(a)



top loop

bottom loop

$$\vec{B}_T = \frac{\mu_0 I}{2} R^2 \left[\frac{1}{\left(R^2+\left(z-\frac{d}{2}\right)^2\right)^{3/2}} + \frac{1}{\left(R^2+\left(z+\frac{d}{2}\right)^2\right)^{3/2}} \right] \hat{z}$$

(b) $$\frac{\partial B_z}{\partial z} = \frac{\mu_0 I R^2}{2} \left[\frac{-\frac{3}{2} 2\left(\cancelto{0}{z}-\frac{d}{2}\right)}{\left(R^2+\left[z-\frac{d}{2}\right]^2\right)^{5/2}} + \frac{-\frac{3}{2} 2\left(\cancelto{0}{z}+\frac{d}{2}\right)}{\left(R^2+\left[z+\frac{d}{2}\right]^2\right)^{5/2}} \right]$$

at $z=0$, terms cancel

$= 0$

(c) $$\frac{\partial^2 B_z}{\partial z^2} = -\frac{3\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2+\left[z-\frac{d}{2}\right]^2\right)^{5/2}} + \frac{1}{\left(R^2+\left[z+\frac{d}{2}\right]^2\right)^{5/2}}\right]$$

$$-\frac{3\mu_0 I R^2}{2}\left[\frac{-\frac{5}{2}2\left(z-\frac{d}{2}\right)^2}{\left(R^2+\left[z-\frac{d}{2}\right]^2\right)^{7/2}} + \frac{-\frac{5}{2}2\left(z+\frac{d}{2}\right)^2}{\left(R^2+\left[z+\frac{d}{2}\right]^2\right)^{7/2}}\right]$$

$\underline{z=0}$

$$= -\frac{3\mu_0 I R^2}{2}\left\{\frac{2}{\left(R^2+\frac{d^2}{4}\right)^{5/2}} - \frac{5\left(-\frac{d}{2}\right)^2+5\left(\frac{d}{2}\right)^2}{\left(R^2+\frac{d^2}{4}\right)^{7/2}}\right\}$$

$$= -\frac{3\mu_0 I R^2}{\left(R^2+\frac{d^2}{4}\right)^{5/2}}\left\{1 - \frac{\left(5/4\right)d^2}{\left(R^2+\frac{d^2}{4}\right)}\right\}$$

$$= -\frac{3\mu_0 I R^2}{\left(R^2+\frac{d^2}{4}\right)^{5/2}}\left\{\frac{R^2+d^2/4-5d^2/4}{\left(R^2+d^2/4\right)}\right\}$$

if $R^2-d^2=0 \Rightarrow \frac{\partial^2 B_z}{\partial z^2}=0$

$$\boxed{R=d \Rightarrow \frac{\partial^2 B_z}{\partial z^2}=0}$$