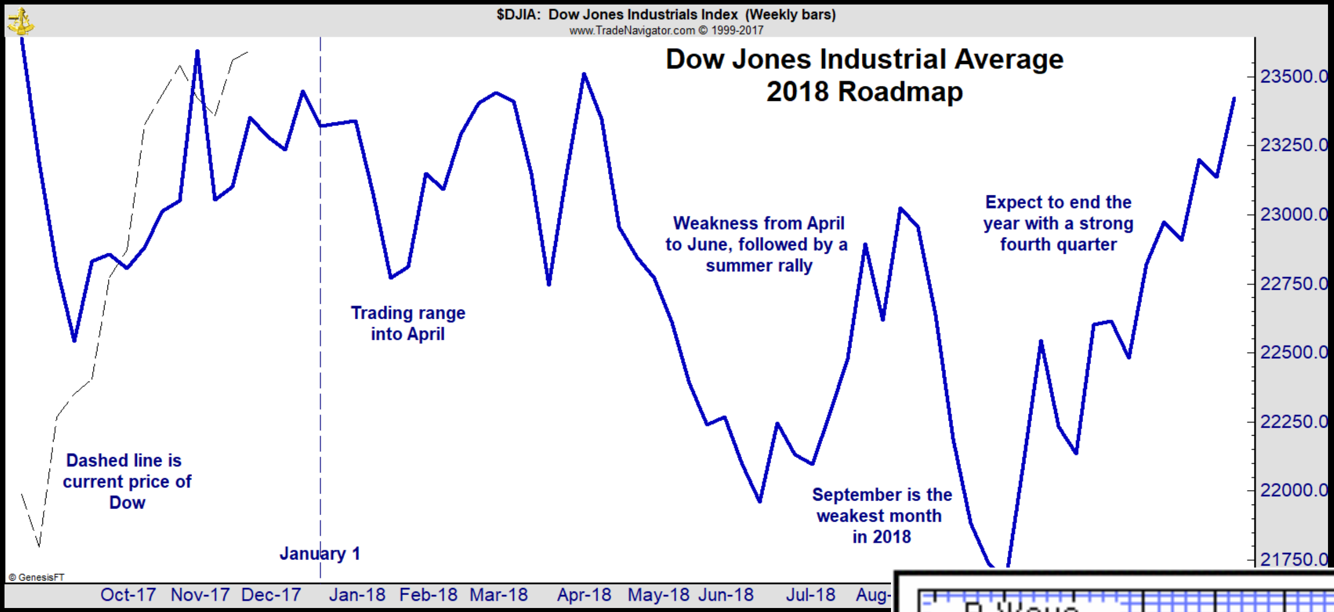
Time Series Analysis

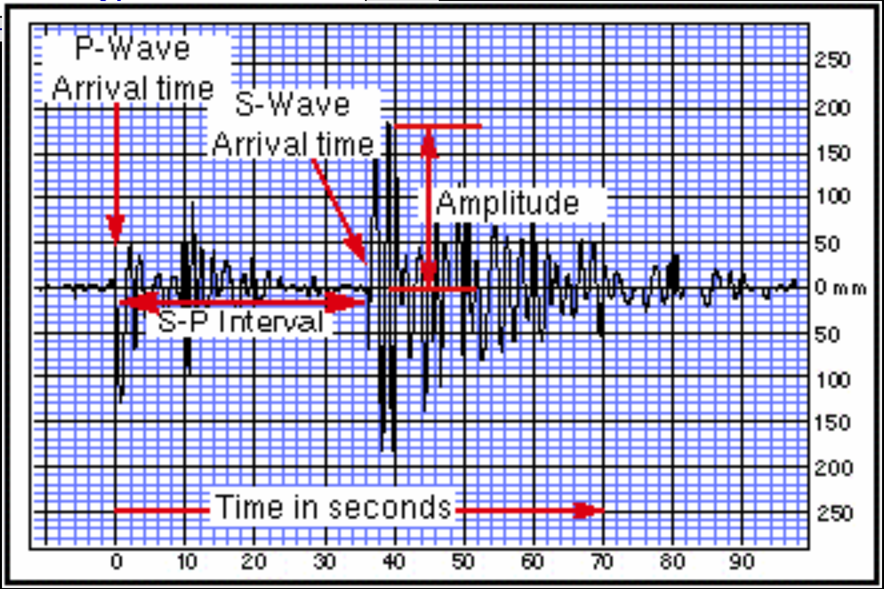
Extract rotation period and/or orbital period from time series data of asteroids

Samples of Time Series Data

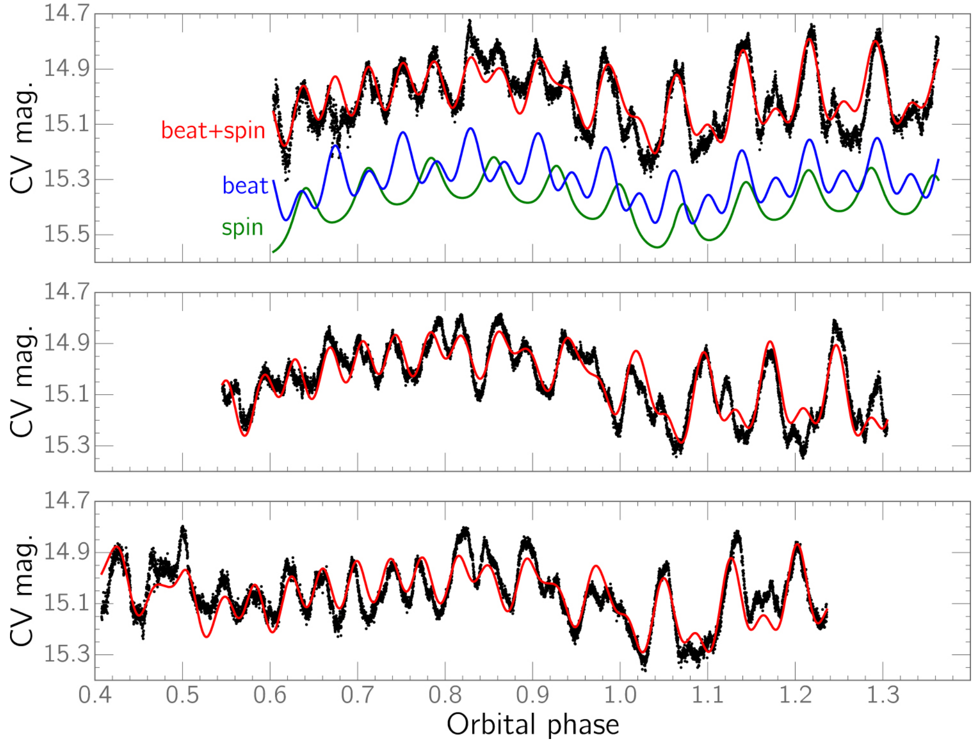


Stock Market

Seismic Signal from Earthquake

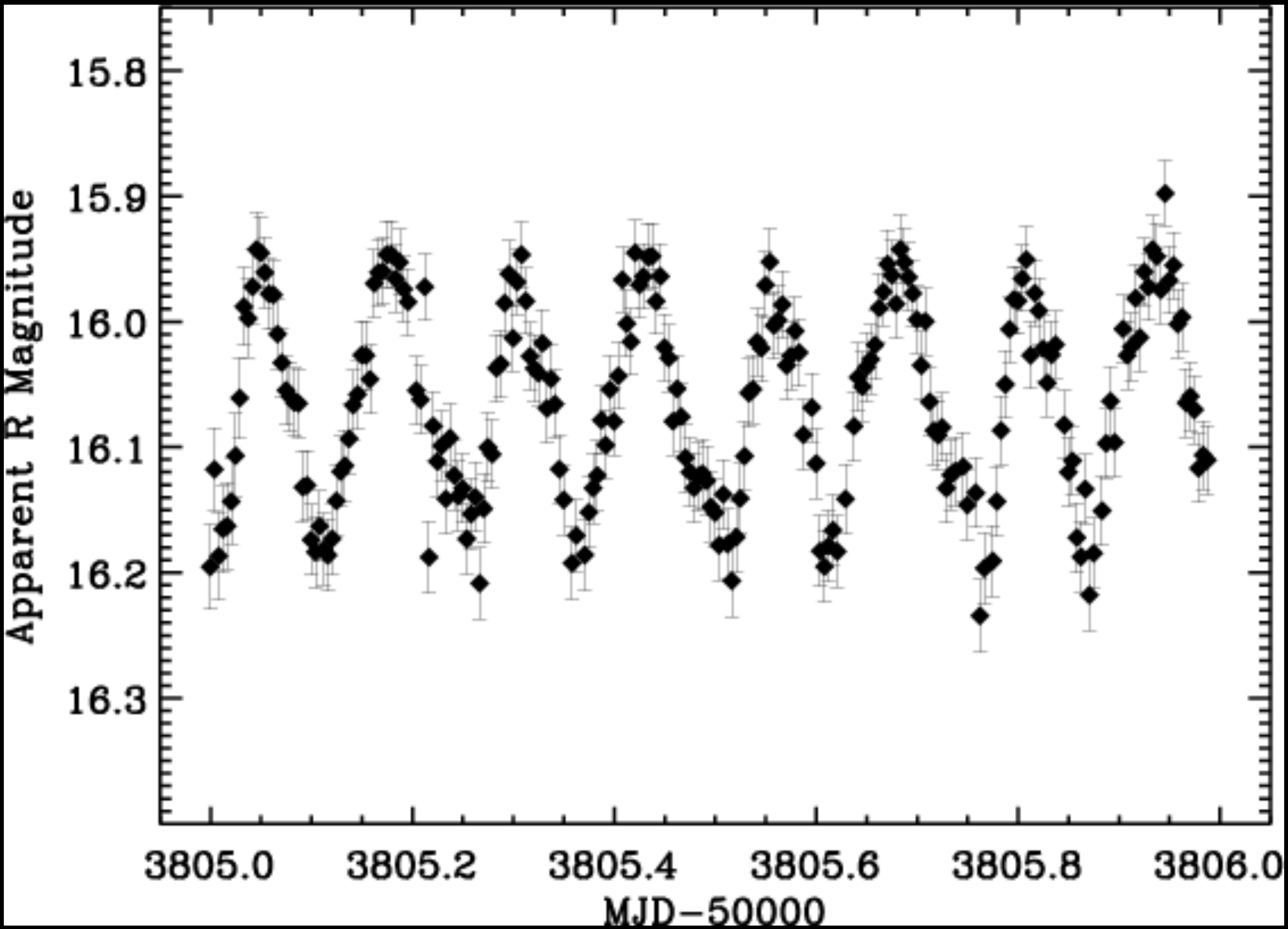


FO Aquari: “King” of the IPs



$$\mathrm{HJD} = 2{,}444{,}782.9169(2) + 0.014519029(1)E + 7.45(4) \times 10^{-13}E^2 - 1.639(7) \times 10^{-18}E^3. \quad (1)$$

One Day of Data



Fourier Analysis

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

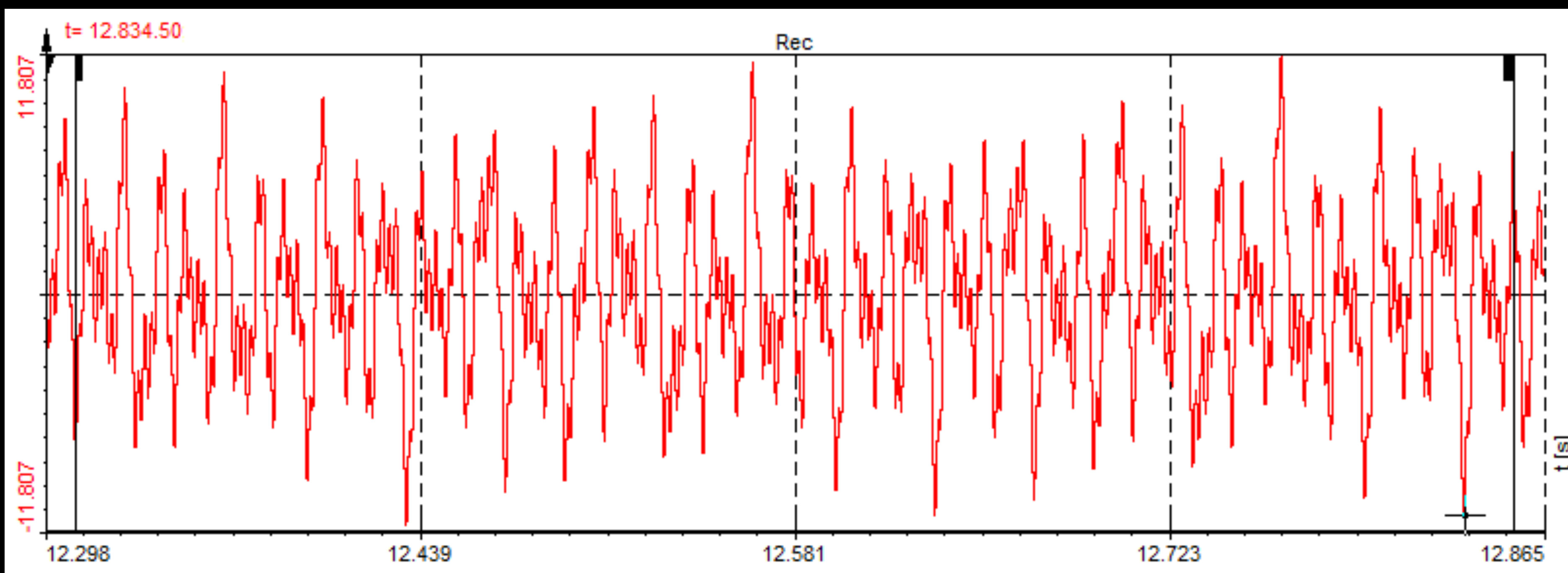
The above is a Fourier series representation of the periodic function g(t)

Properties of Fourier Series

- Linearity
- Multiplication
  - Multiplication in time leads to convolution of FS
- Time Shifting
  - Time shift leads to linear phase shift in FS
- Time Reversal
  - Time reversal leads to index reversal
- Time Scaling
  - Time scaling leads to frequency stretching
- Conjugation: $x_p^*(t) \Leftrightarrow \{c_{-n}^*\}$
- Parseval's Relation:
  - Energy contained in FS

$$\frac{1}{T_0}\int_{T_0} |x_p(t)|^2 \, dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

- Fitting a Fourier series to long strings of unevenly spaced data is straightforward, but inefficient from a computational standpoint. The work scales as N^2 where N is the number of data points
- When the data points are evenly spaced, the numerical problem has a clever solution, the so-called the FAST FOURIER TRANSFORM (FFT) which, IMO, is the computer algorithm of the last 100 years. The FFT scales as $N \log_e N \ll N^2$ for large N.
- Application of the FFT to the data below allows a rapid decomposition of the time series showing that a seemingly random process is actually a linear combination of a small number of waves



The time series g(t) represented by the Fourier series

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

is analyzed as follows

The goal in Fourier analysis is to determine the coefficients a_m and b_n and so find how the different waves add and mix to form the signal.

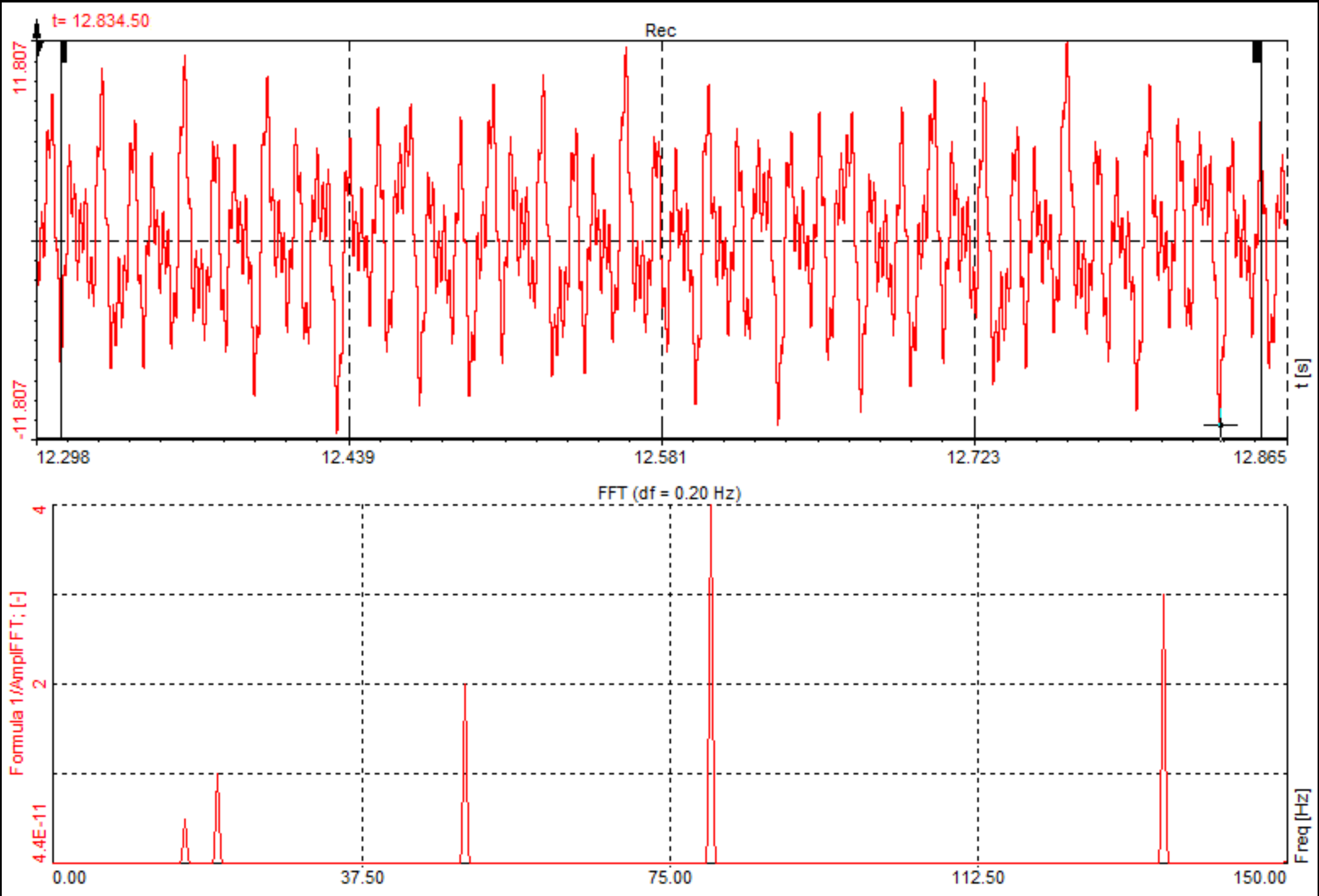
This is accomplished by using the fact sine and cosine functions are orthogonal over the range $t = [0,T]$.

$$\int_{-L}^{L} \cos\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi x}{L}\right)dx = \begin{cases} 2L & \text{if } n = m = 0 \\ L & \text{if } n = m \neq 0 \\ 0 & \text{if } n \neq m \end{cases}$$

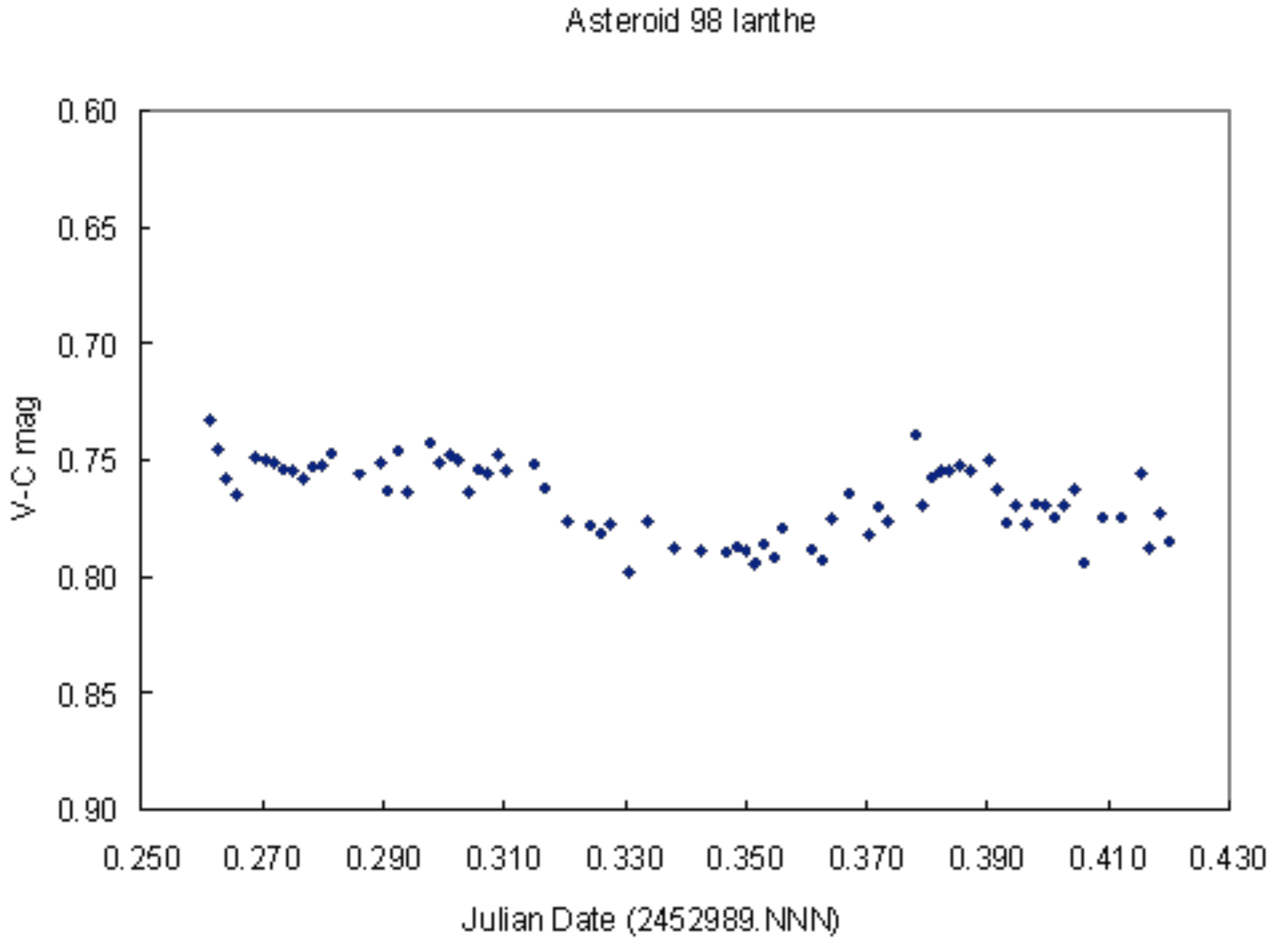
$$\int_{-L}^{L} \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = \begin{cases} L & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases}$$

$$\int_{-L}^{L} \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi x}{L}\right)dx = 0$$

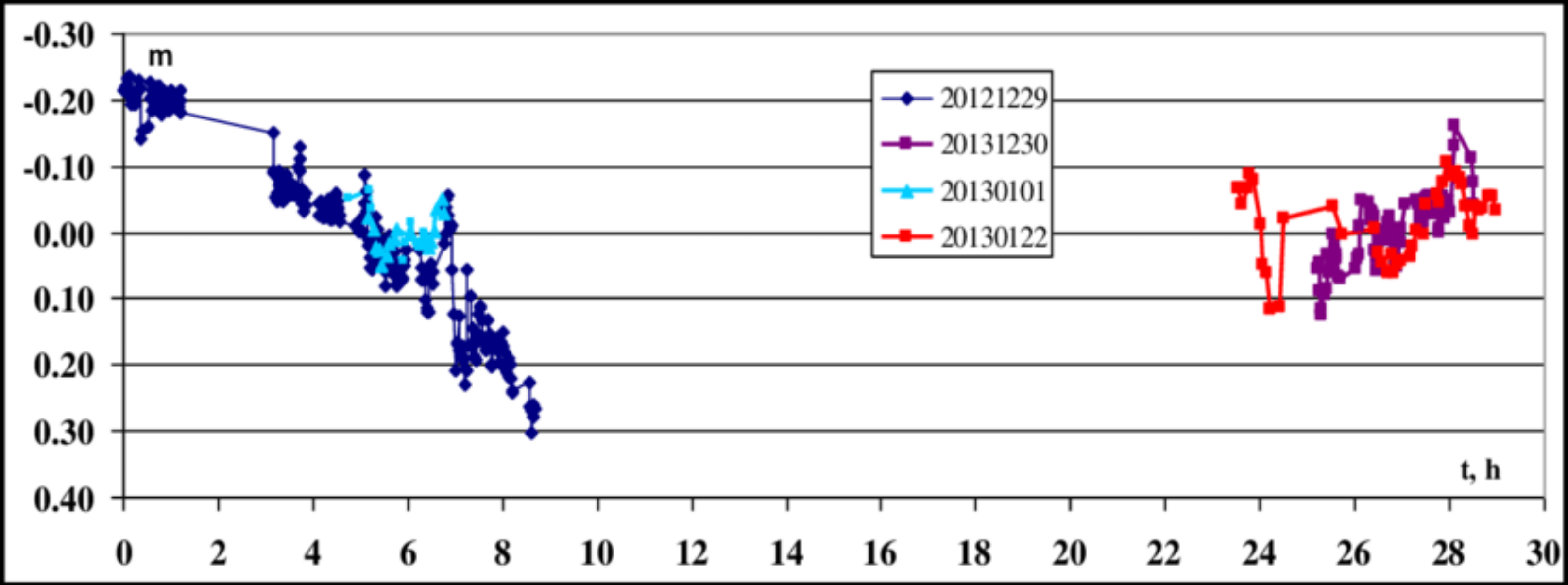
Note: *Let L = T and x = t* to recover our formulation.

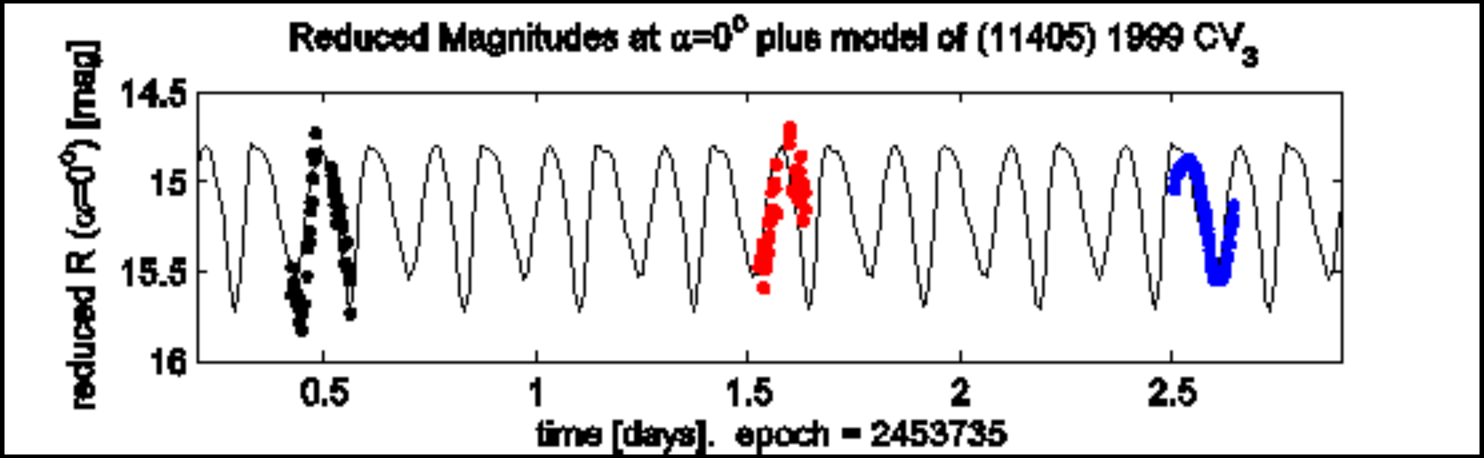

t= 12.834.50
Rec
11.807
-11.807
t [s]
12.298
12.439
12.581
12.723
12.865
FFT (df = 0.20 Hz)
4
2
4.4E-11
Formula 1/AmplFFT; [-]
Freq [Hz]
0.00
37.50
75.00
112.50
150.00

More Typical Single Observation

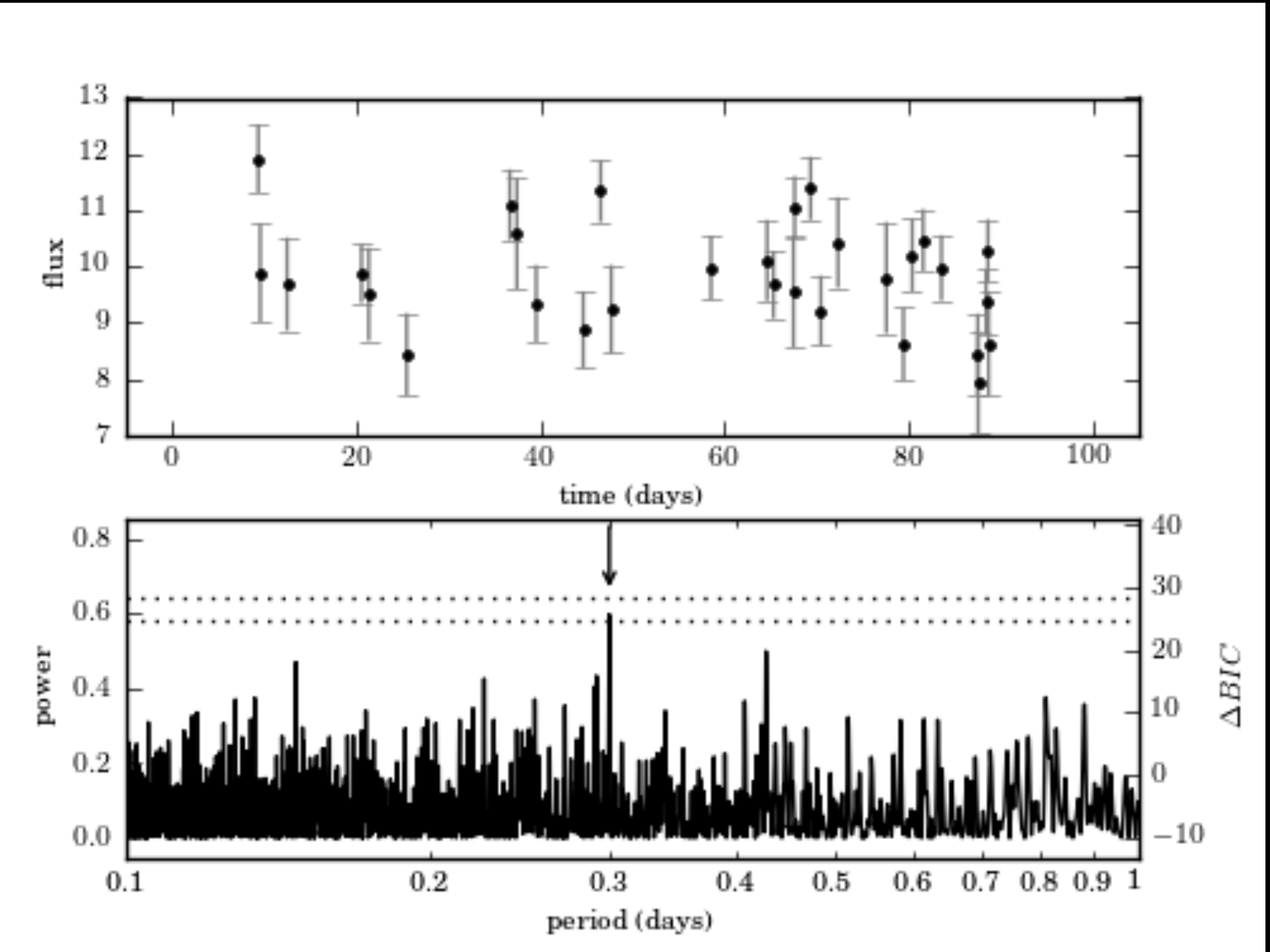


Several Nights of Data





Unevenly Spaced and Sparse Data: Use Lomb-Scargle Periodogram

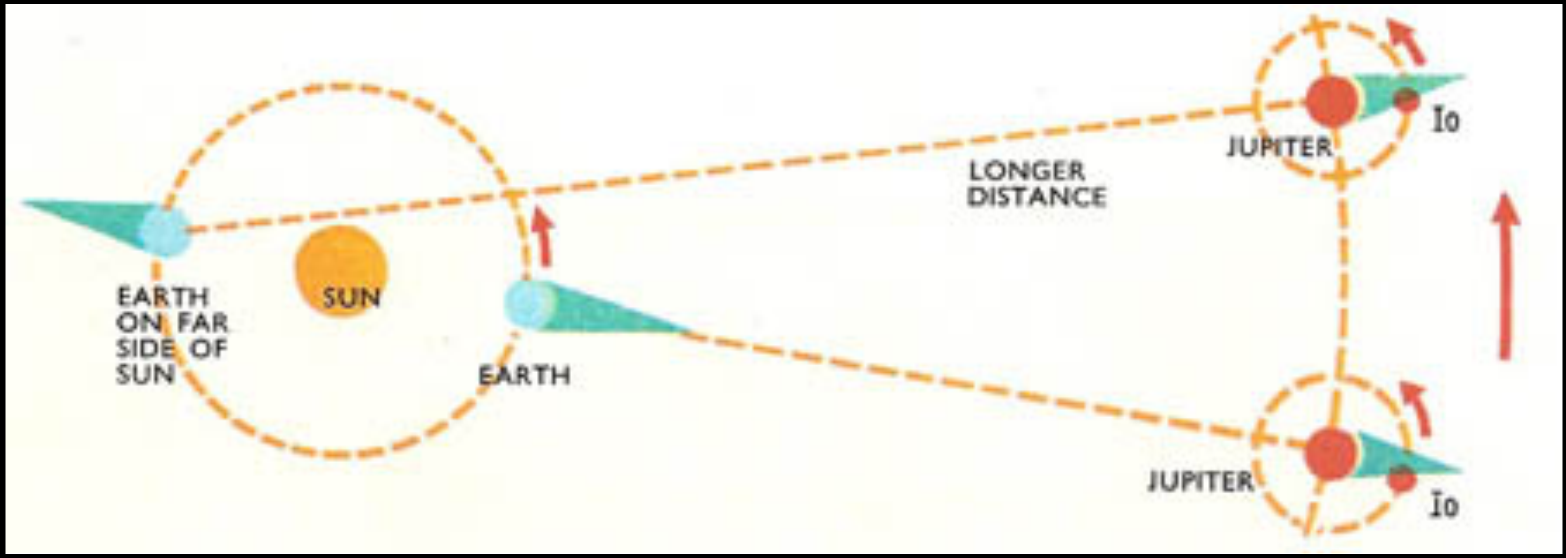


Complications

- Viewing Aspect
- Time System
- Orbit-to-Orbit variation
- Rotation-to-Rotation variation
- …

Time Systems?

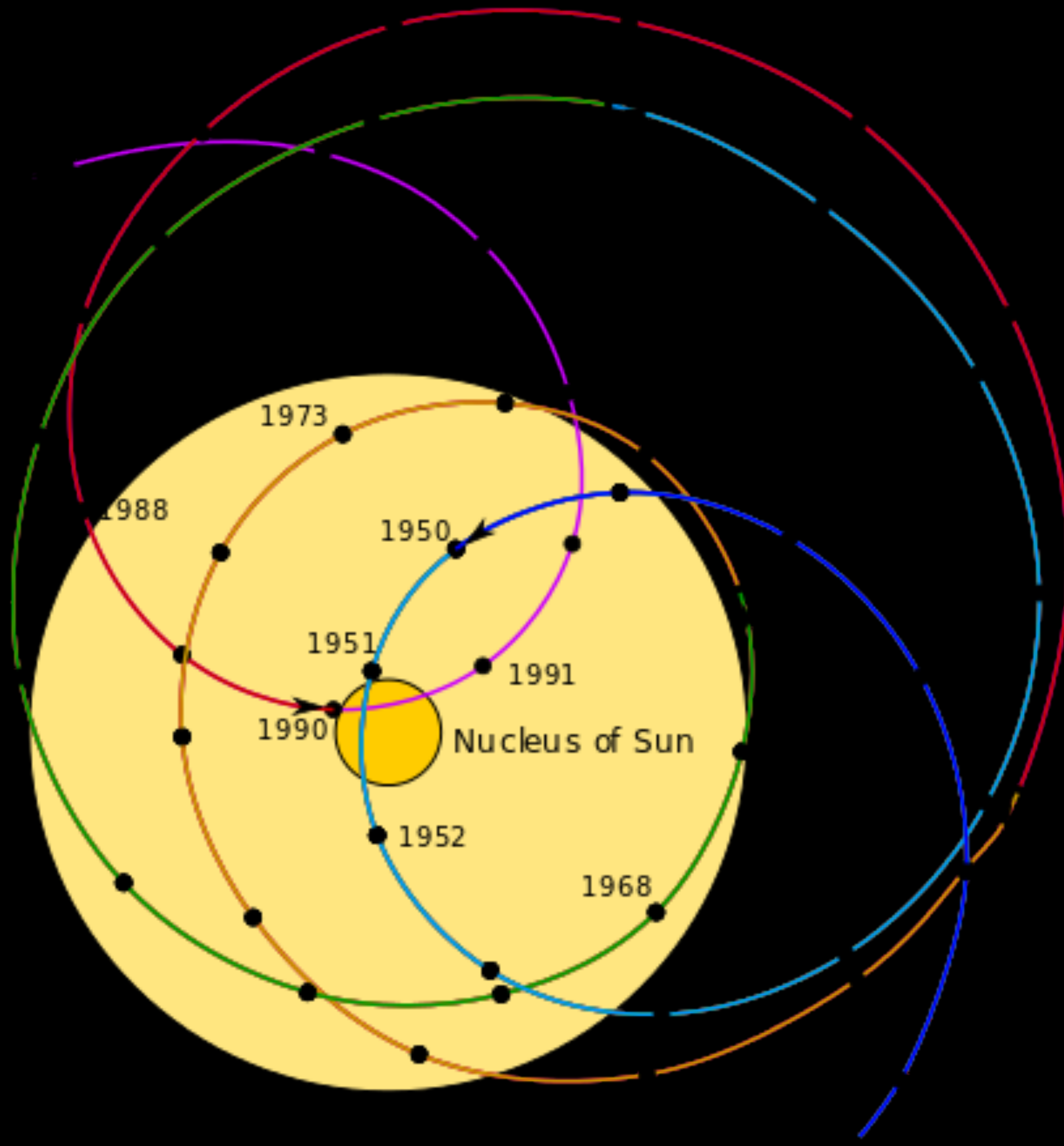
- Which system to use?
- Actually a tricky problem because when stringing data taken on different days to form one time series, how far away is the asteroid from the Earth affects the light travel time between the asteroid and us, and so affects how long it takes a pulse of light to reach us.

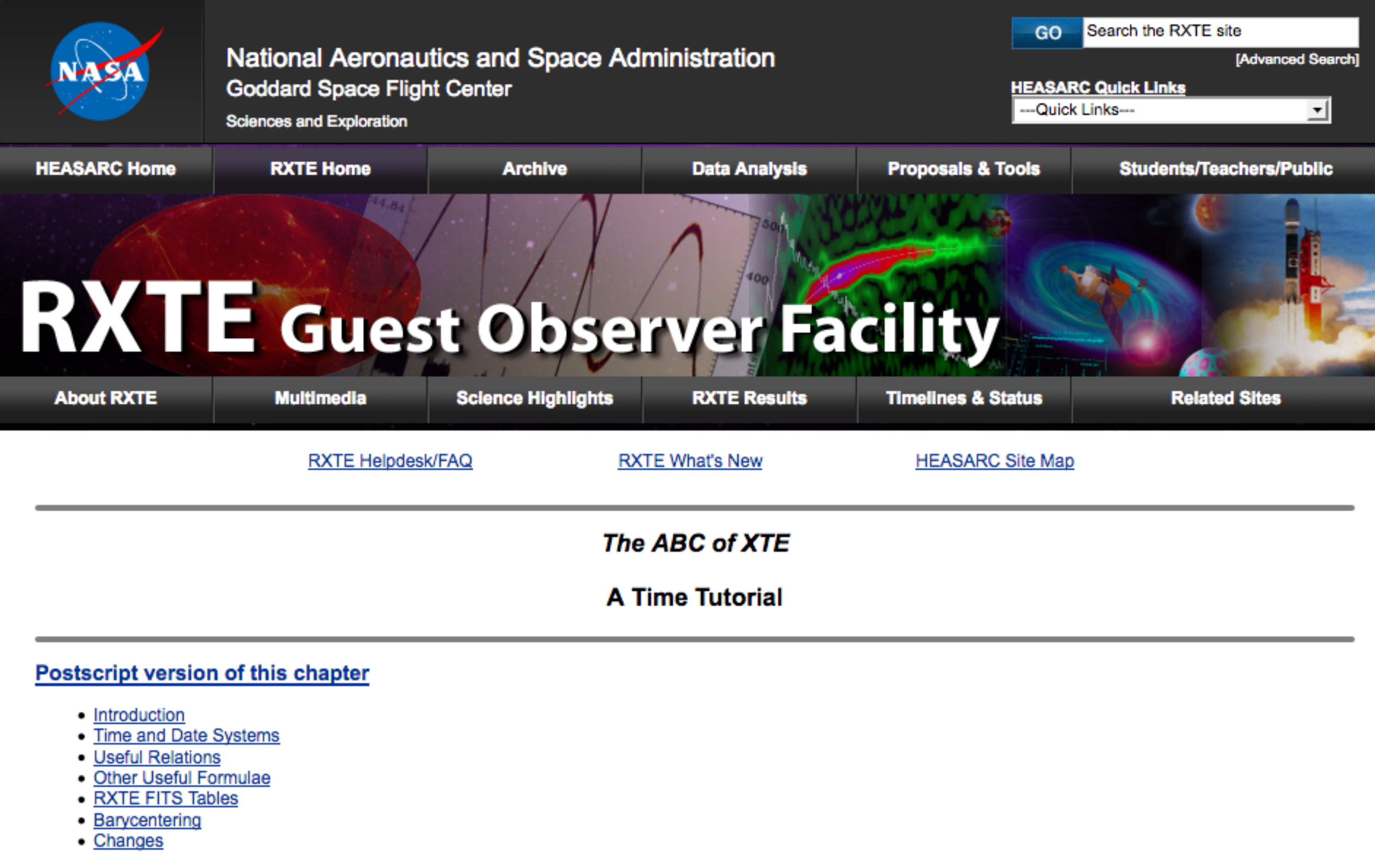


- Not a problem over the course of a night, errors of only up to ~0.05 s or so may arise.
- It could be a problem if one wants to string data from several nights separated by weeks together. To overcome this, we convert all times to some standard frame, the barycentric dynamical time TDB, times as if measured at the barycenter of the Solar System

Barycenter corresponds to the balance point

Motion of the barycenter of the Solar System with respect to the Sun*





https://heasarc.gsfc.nasa.gov/docs/xte/abc/time_tutorial.html

- TAI (International Atomic Time): Based on the SI second and derived from a large number of clocks all over the world. Analyzed and published by by BIPM. Reduced to mean sea level.
- UTC (Universal Time, Coordinated): Based on the SI second, it knows days of 86400 and 86401 seconds. The latter are days with a so-called leap second. The objective is to keep UTC within a second of UT1, which is based on the earth's rotation. The offset from TAI is always an integer number of seconds (the accumulated leap seconds).
- TT (Terrestrial Time): An idealized time, for all practical purposes based on TAI, but with a constant offset to provide continuity with Ephemeris Time. Defined on the rotating geoid.
- TB (Barycentric Time): Also known as TDB (Barycentric Dynamical Time). This is basically TT transformed to the solar system barycenter.
- JD (Julian Date): Number of days since Greenwich mean noon on January 1, 4713 B.C. Note that JD may be used in conjunction with all of the above time systems; whenever the required accuracy dictates this, the time system used should be indicated: e.g., JD (TT).
- MJD (Modified Julian Date): Equals JD - 2400000.5. The same note applies as for JD concerning the time system in use. We shall continue to use MJD notwithstanding the IAU's recent decision not to recognize MJD anymore as an official time unit.