1.1 - Functions

$\boxed{\text{Def}}$ A relation is a pairing of two sets of elements, the first group of inputs called the domain, the second group of outputs called the range of the relation. A pair of elements, say x from the first set and y from the second set, can be written as (x, y).

Ex 1 Write the relation that pairs a whole number with the square of that number.

$\boxed{\text{Def}}$ A function is a relation in which each element of the first group corresponds to exactly one element from the second group (i.e. each input corresponds to exactly one output). Using the notation from the definition of a relation above, we say that "y is a function of x".

Ex 2 Does the relation that associates the cost of a book with its title define a function?

Ex 3 Given the data in the following table...

x	1	6	8	11	14
y	-1	4	6	4	0

a) What are the domain and range of the relation with elements (x, y)?

b) Does the table define y as a function of x? Explain.

c) Does the table define x as a function of y? Explain.

Ex 4 Find a function defined from a table such that the domain of the function is $\{-2, -1, 0, 1, 2\}$ and the function associates each input with its absolute value. What is the range of this function?

$\boxed{\text{Def}}$ (Function Notation)
If y is a function of x, we can name that function, say f, and rewrite $y = f(x)$.
$f(x)$ is read "f of x", and is not the same as "f times x".

Ex 5

a) Write the function from Example 1 using function notation. Name the function f.

b) Find the value of $f(2)$ and $f(16)$.

Ex 6

a) Write the function from Example 4 using function notation. Name the function g.

b) Find the value of $g(-2)$ and $g(3)$.

Ex 7 Consider the function $f(t) = \sqrt{2t - 1}$.

a) Find and simplify $f(5)$.

b) What is the largest possible domain of f? Write your answer using interval notation.
(See section 0.3 for a brush-up on interval notation)

Ex 8 Consider the function $h(x) = \dfrac{2-3x}{|x+2|-3}$.

a) Compute $h(-4)$ and $h(1)$.

b) What is the largest possible domain of h? Write your answer using interval notation.

c) Find all real solutions to $h(x) = 1$.

Ex 9 Consider the function $m(x) = |x-4|$.

a) Find and simplify $m(\pi)$. Write your answer without absolute value signs.

b) What is the largest possible domain of m? Write your answer using interval notation.

c) Find all values of x such that $m(x) < 1$. Depict this solution set on the number line.

$\boxed{\text{Def}}$ The domain convention is the assumption that, when a domain is not explicitly given for a function, the domain is the part of the real numbers for which the function defines a real number. (i.e. the function is defined for all real numbers that "make sense")

$\boxed{\text{Def}}$ A piecewise-defined function is a function defined from several pieces of functions, each of which is defined only on a specific domain.

Ex 10 Consider the function $f(x) = \begin{cases} x^2-1 & \text{, if } x \leq -2 \\ \sqrt{x+8} & \text{, if } -2 < x < 1 \\ 2x+1 & \text{, if } x \geq 1 \end{cases}$.

a) What is the domain of f? Write your answer in interval notation.

b) Compute and simplify $f(0)$, $f(-3)$, and $f(|x-3|+2)$.

c) Find two different values of x for which $f(x) = 9$.

$\boxed{\text{Def}}$ Two functions are equal exactly when (1) their domains are equal and (2) they evaluate to the same elements in the range for each number in the domain.

Ex 11

a) Are the functions $f(x) = 3x+2$, on domain $\{1,2\}$, and $g(x) = x^2+4$ on domain $\{1,2\}$ equal?

b) Find the value of a so that $f(x) = ax+3$ and $g(x) = \dfrac{x+3}{x+1}$ are equal on domain {0,3}.

Ex 12 Define the functions $f(x) = |3x-2|$, $g(x) = x^2-3x+1$, and $h(x) = \dfrac{\sqrt{x+3}}{5-x}$.

a) Find and simplify $f(-1) - g(-2) + h(1)$.

b) Find all values of x such that $g(x) = 11$.

c) Find all values of x such that $f(x) \geq 2$. Write your answer in interval notation.

d) Find and simplify $\dfrac{g(a-3)-g(-3)}{a}$, assuming $a \neq 0$.

e) Write the domain of h using interval notation.