Math 252 Calculus II

Syllabus 2012-13 [written by Arkady Vaintrob]

Prerequisite: Math 251 or equivalent.

Text: *Calculus, Concepts and Contexts, 4th edition*, by James Stewart.

We will cover Chapters 5—7 of the book *except* Section 6.8. The material of the omitted Section 6.8 on probability will be studied in Math 253.

Course description and goals

This is the middle part of the Calculus sequence. First and foremost this course is about definite integrals—their meaning as the limit of sums, their relationship with derivatives (the Fundamental Theorem of Calculus), the basic integration techniques (primarily the substitution and integration by parts), and their applications in mathematics and other disciplines. A significant part of the course is devoted to differential equations.

The course has three major themes which roughly correspond to its structure.

- *Theoretical foundations of the integral calculus (Sections 5.1–5.5 and 5.10)*

In this part of the course students will learn that the integral is defined as the limit of sums (and, like every limit, it may or may not exist) and that it is related to the derivatives via the Fundamental Theorem of Calculus. It is very important that the students understand the meaning and implications of this result. In particular they should be absolutely clear that, even though the integral of a continuous function can be *computed* using its antiderivative, it is not how it is *defined*. Specifically, students should know how to write Riemann sums for a given function, compute integrals of simple functions directly from definition, and interpret integrals in terms of areas, as well as to use upper and lower sums to estimate integrals. They should be able to use the definition of the integral to express various quantities involving limits of sums.

- *Techniques of integration (Sections 5.5–5.9)*

The second part of the course is devoted to various methods of evaluating integrals. Students should know the list of standard antiderivatives and how to use tables to find antiderivatives of more involved functions. They should be able to use the substitutions and integration by part techniques to compute basic integrals. They should also be familiar with the method of partial fractions for integrating simple rational functions. More specific methods of indefinite integration (e.g. for rational and trig functions or functions involving roots of quadratic polynomials) will also be discussed but not overemphasized.

Section 5.8 on integration using tables and computer algebra system is optional. It may be treated in passing or omitted completely. Section 5.9 should also be treated lightly. The goal here is to make sure that the students are aware that many elementary functions (in fact, the majority of them) do not have an elementary antiderivative, and so one needs to use other tools (such as estimates or approximate integration) to deal with integrals of such functions.

- *Applications of integrals and differential equations (Chapters 6 & 7)*

  The main bulk of the course concerns applications of integrals in mathematics and other disciplines. The goal for students is to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral. The students are supposed to learn how to use integrals to compute quantities appearing in "practice", such as areas, volumes, lengths, averages, estimates of sums, etc. Besides these "mathematical" applications, students should be able to use integrals to model "real life" situations arising in the natural and social sciences. This means doing plenty of word problems.

  A substantial part of the course is devoted to an introduction to differential equations, which is one the most important of all the applications of calculus. Again the main theme here is using differential equations to model physical and biological processes. Since the majority of differential equations arising in practice cannot be solved explicitly, students should be able to use qualitative, numerical and analytic methods.

Exams: There will be 2 midterms, tentatively during 4th and 7th weeks [roughly focused on evaluation of integrals and their applications, respectively] and a comprehensive final exam.

Homework: There will be weekly homework assignments.

Quizzes: There may be quizzes

Tentative Schedule

Week 1	5.1–3	Riemann sums, definite and indefinite integrals
Week 2	5.4–6	Fundamental Theorem, Substitutions, Integration by parts
Week 3	5.6–8 (5.8 optional)	Integration techniques
Week 4	5.9–10, Exam 1	Approximate integration, Improper integrals
Week 5	6.1–3 (6.3 optional)	Areas, volumes
Week 6	6.4–6 (6.4 optional)	Arc length, averages, physics applications
Week 7	6.7, 7.1, Exam 2	Biology/economics applications, Differential equations
Week 8	7.2–3	Direction fields, Separable equations
Week 9	7.4–6 (7.6 optional)	Applications of differential equations
Week 10	Catch-up and review	

There will not be enough time (nor need) to do all the applications from Sections 6.6 and 6.7. Their selection should be based on the interests of the students and the instructor.