Math 251
Final Exam - Winter 2013

Name:_____ **Student ID #:** _____

You will be asked to present your student ID when you turn in your final and may be asked to show your ID at other times during the exam.

Instructions:

- You must show work that justifies your answer to receive credit and all notation must be correct.
- No electronic devices may be used.
  This includes calculators, phones, electronic dictionaries, headphones, etc.
- Please silence your cell phone.
  If your phone is a distraction to others, you may lose points on your exam.
- Check your work once you have worked through all of the problems.

1	(12)	
2	(12)	
3	(6)	
4	(7)	
5	(6)	
6	(8)	
7	(10)	
8	(9)	
9	(12)	
10	(6)	
11	(12)	
Total	(100)	

(1) (12 pts) Compute the following limits. In some cases the correct answer may be $\infty, -\infty$ or the limit may not exist.

You must justify your answer: no credit will be given if you just state the limit.

(a) $\lim_{x\to-\infty} \dfrac{2-7x+2x^2-4x^4}{5x^4+2x+1}$

(b) $\lim_{x\to\infty} \dfrac{x^2}{e^x}$

(c) $\lim_{u\to 0} \dfrac{u^2}{\cos u}$

(d) $\lim_{t\to 1} \dfrac{5^t-5}{t-1}$

(2) (12 pts) Find the derivative of each function.

(a) $f(x) = \dfrac{\cos(x)}{\sqrt{x^2+1}}$

(b) $g(x) = \ln(b(x) - x)$
Here $b(x)$ satisfies $b'(x) = -5b(x) + 1$ and your answer may involve the function $b(x)$.

(c) $h(x) = \arctan(kx - 1)$
(k is constant.)

(d) $A(u) = \dfrac{4}{3}\pi(u^2 - 6)^2 + \pi^2$

(3) (6 pts) Let $y = x^x$. Using logarithmic differentiation, find $\dfrac{dy}{dx}$ in terms of x.

(4) (7 pts) A ballon is rising from the ground. When it is 30 meters above the ground, it is rising at a rate of 3 meters per second. An observer is on the ground 40 meters from the point where the balloon originated.

At what rate is the balloon moving away from the observer at this moment when the balloon is 30 meters above the ground? Include units in your answer.

(5) (6 pts) Find the equation of the tangent line to the curve $2x^2 + y^3 = 3$ at the point $(1, 1)$.

(6) (8 pts) A farmer plans to build a rectangular pasture next to a river. The pasture must contain $180,000$ square meters in order to provide enough grass for the herd. What dimensions require the least amount of fencing if no fencing is needed along the river?

(7) (10 pts) Consider the function $f(x) = x^5 - 5x + 1$.

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local maximum and minimum values of f.

(c) How many solutions does $x^5 - 5x + 1 = 0$ have? Explain why your answer is correct.

(8) (9 pts) An ant is walking along the edge of a ruler. Its position in cm along the ruler at time $t = 1$ to $t = 4$ is given by

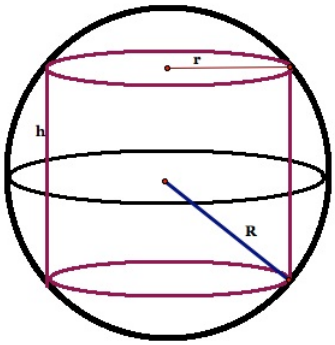
$$s(t) = t^3 - 6t^2 + 9t + 1$$

(a) At what moments of time is the ant at rest?

(b) When is the ant the furthest along the ruler?

(c) What is the total distance the ant walks between $t = 1$ and $t = 4$?

(9) (12 pts) A cylinder is inscribed in a sphere of radius $R = 10$. Find the largest possible volume of such a cylinder.



(10) (6 pts) Find the absolute maximum and minimum of $f(t) = 2t^3 - 9t^2 + 12t - 1$ on $[1, 3]$.

(11) (12 pts) Consider the function

$$f(x) = \frac{x^2 - 2x + 4}{x - 2}.$$

Sketch the graph $y = f(x)$ by finding all asymptotes, intervals of increasing and decreasing, local minima and maxima, intervals of concavity, and inflection points.