REFERENCE ON K-THEORY

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This is a collection of references on K-theory. I expect to add more; these are the ones I know about now, gotten from various sources.

The comments vary greatly in how informative and how accurate they are.

As far as I know, E-theory (KK-theory via asymptotic morphisms) is only in (2) (at the end). (This is the version of KK-theory I used for the purely infinite classification theorem, but is grossly unsuited to index theory.) Extension theory (starting with the Brown-Douglas-Fillmore version of K-homology) is only in (2) and (4). The reference (8) is included mainly for its proof of Bott periodicity.

There is nothing on algebraic K-theory, because I don't know the books in the area. There certainly are some.

(1) M. F. Atiyah, *K-Theory*, W. A. Benjamin, New York, Amsterdam, 1967.

The original book on topological K-theory. Considered old fashioned.

(2) B. Blackadar, *K-Theory for Operator Algebras*, 2nd ed., MSRI Publication Series **5**, Cambridge University Press, Cambridge, New York, Melbourne, 1998.

The original book on K-theory of C*-algebras. Comprehensive; condensed. Has several chapters on equivalence relations on projections and unitaries, and how the relations are related to each other, before K-theory, and the reason for these relations, is even mentioned.

(3) A. Hatcher, *Vector Bundles and K-Theory*, Version 2.2, November 2017, available at `https://pi.math.cornell.edu/~hatcher/VBKT/VB.pdf`, via `https://pi.math.cornell.edu/~hatcher/VBKT/VBpage.html`.

Book in preparation. According to the web page, about half done, but most of the more basic material is there. No operator algebras.

(4) N. Higson and J. Roe, *Analytic K-Homology*, Oxford Mathematical Monographs. Oxford Science Publications. Oxford University Press, Oxford, 2000.

K-theory (review), K-homology via both extensions and abstract elliptic operators, but very little about KK-theory. It contains introductory material on C*-algebras for readers not familiar with them.

(5) K. K. Jensen and K. Thomsen, *Elements of KK-Theory*, Birkhäuser, Boston, Basel, Berlin, 1991.

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As far as I recall, this book assumes C*-algebras, and roughly follows Kasparov's approach, via abstract elliptic operators. I know little about this book.

(6) M. Karoubi, *K-Theory. An Introduction*, Grundlehren der Mathematischen Wissenschaften no. 226, Springer-Verlag, Berlin, New York, 1978.

Topological K-theory, using a lot of algebraic machinery. Treats a number of more advanced topics, including the Chern character, for which I have cited it as a reference.

(7) E. Park, *Complex Topological K-Theory*, Cambridge Studies in Advanced Mathematics, no. 111, Cambridge University Press, Cambridge, 2008.

Topological K-theory done with C*-algebraic machinery, which simplifies some arguments. Several things I consider basic (and which are in Atiyah's book) are missing.

(8) J. L. Taylor, *Banach algebras and topology*, pages 118–186 in: *Algebras in Analysis (Proc. Instructional Conf. and NATO Advanced Study Inst., Birmingham, 1973)*, Academic Press, London, 1975.

This is where I learned about K-theory for Banach algebras. When I was a graduate student, it was the only source. The first few sections are actually about Čech cohomology. Banach algebras are assumed commutative, which is not needed anywhere except in theorems involving maximal ideal spaces. (He passes to matrix algebras, as one must, and of course commutativity is lost.) The description of K_0 is idiosyncratic. Carefully done elementary proof of Bott periodicity for Banach algebras.

(9) N. E. Wegge-Olsen, *K-Theory and C*-Algebras*, Oxford University Press, Oxford etc., 1993.

Leisurely introduction to K-theory of C*-algebras, with introductory material on C*-algebras for those not familiar with them. No KK-theory. Has several chapters on equivalence relations on projections and unitaries, and how the relations are related to each other, before K-theory, and the reason for these relations, is even mentioned.

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