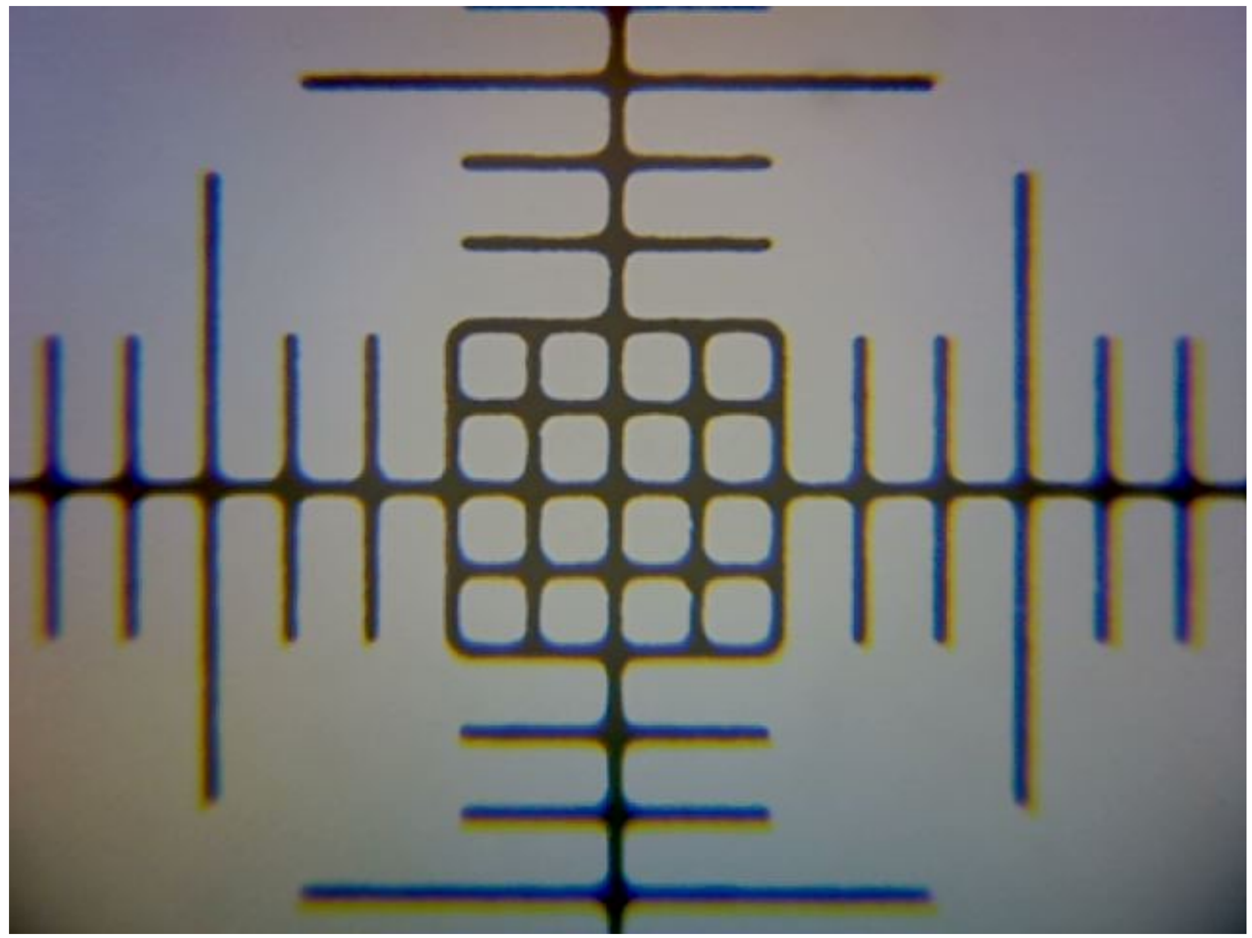
PHYS 391 - Day 12

Lab 3 discussion
Hypothesis Testing

http://pages.uoregon.edu/torrence/391/class/day12.pdf

Lab 3 discussion

Calibration



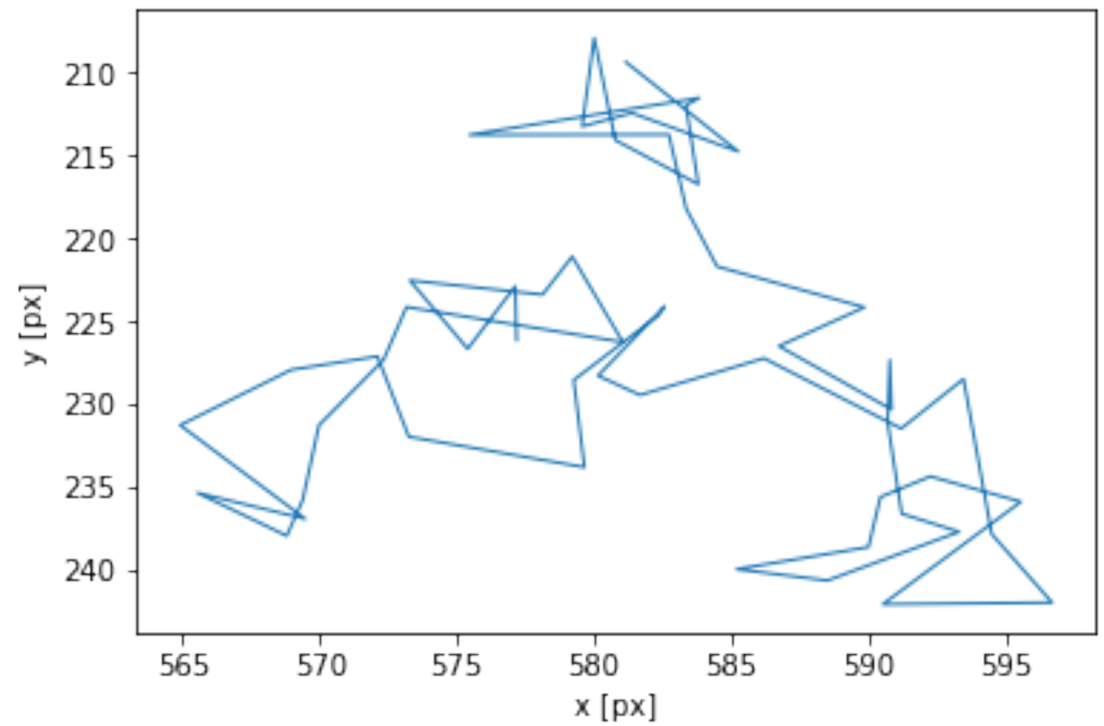
You must calibrate the images measured in microns

Determine µm per pixel

The marks on this slide are 10 microns apart

(assume this has negligible uncertainty)

Particle Tracking



(x_i, y_i) arrays

Do describe briefly the settings used in the trackpy process (mass cut, radius, …) along with the movie you started from (bead diameter, frame rate, x40 or x100)

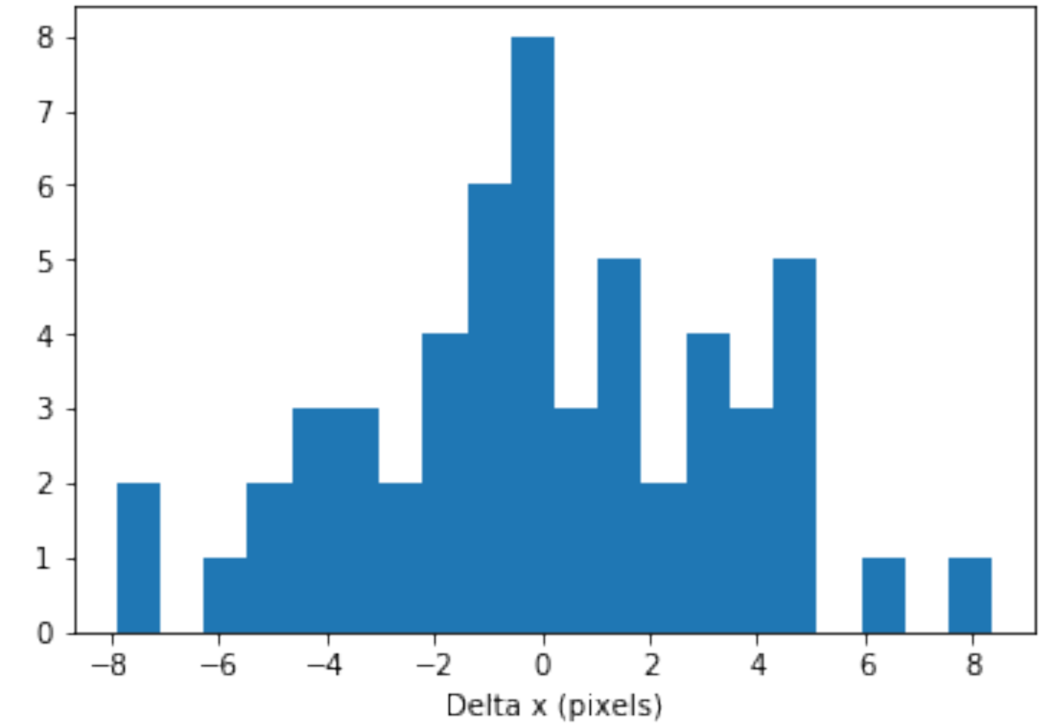
Your analysis starts here

Consistency

(x_i, y_i) arrays

↓

Δx_i and Δy_i arrays →



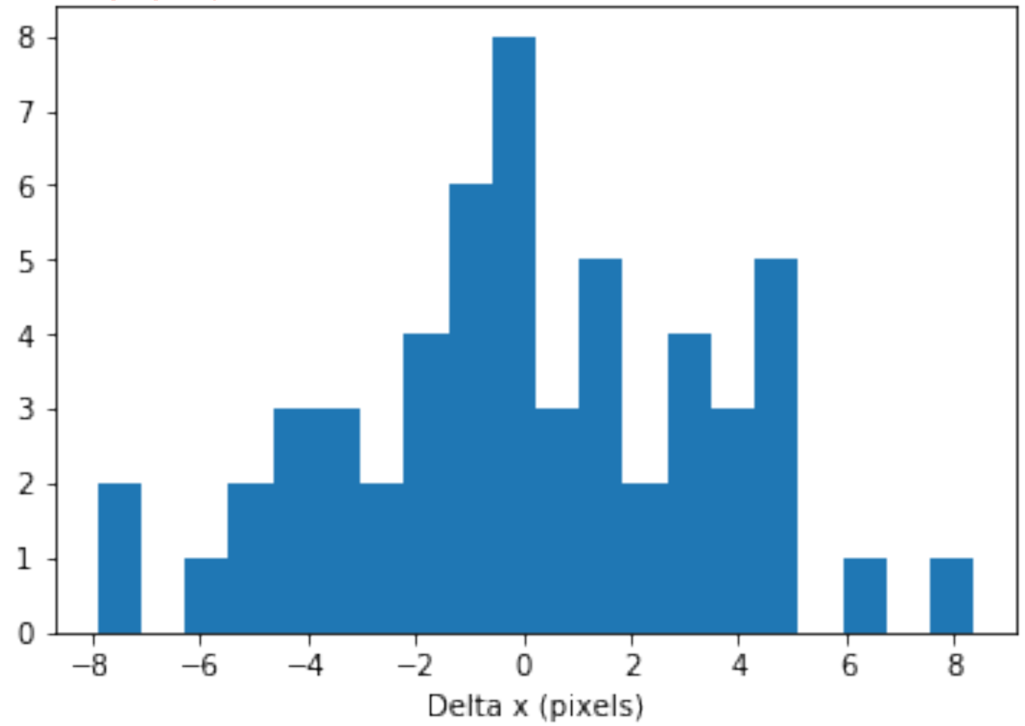
These Δx and Δy distributions should have $\mu = 0$ and $\sigma_x = \sigma_y$. Test this quantitatively!

Remember, we talked about the stat error on the standard deviation! Hwk 2 problem 5.28

Don't worry if you are up to 2 sigma off…

Diffusion

Apply calibration somewhere!



In each dimension:

$$D_x = \overline{\Delta x^2}/(2\Delta t)$$

But since mean is zero:

$$\overline{\Delta x^2} = \sigma_x^2$$

Variance is proportional to diffusion constant - errors

Note: D_x and D_y are measuring the same thing!

Either take average, or measure 2D diffusion:

$$D = (\sigma_x^2 + \sigma_y^2)/(4\Delta t)$$

Boltzmann constant

- Diffusion Coefficient D is your primary experimental observable, make sure you state this clearly with (correct) uncertainties

- Can interpret this as the Boltzmann constant:

$$Df = k_B T \qquad \text{where} \qquad f = 6\pi\eta R$$

Beads have diameters of either 2.54 μm or 1.06 μm

Error propagation problem, straight off HW1, assumptions

Be careful with units! You may be off by x2,
if you are off by 10^{12} you have forgotten to convert microns…

Dependence on Δt

- Diffusion relation predicts variance $\sigma^2 \sim \Delta t$

$$2\Delta t D_x = \overline{\Delta x^2}$$

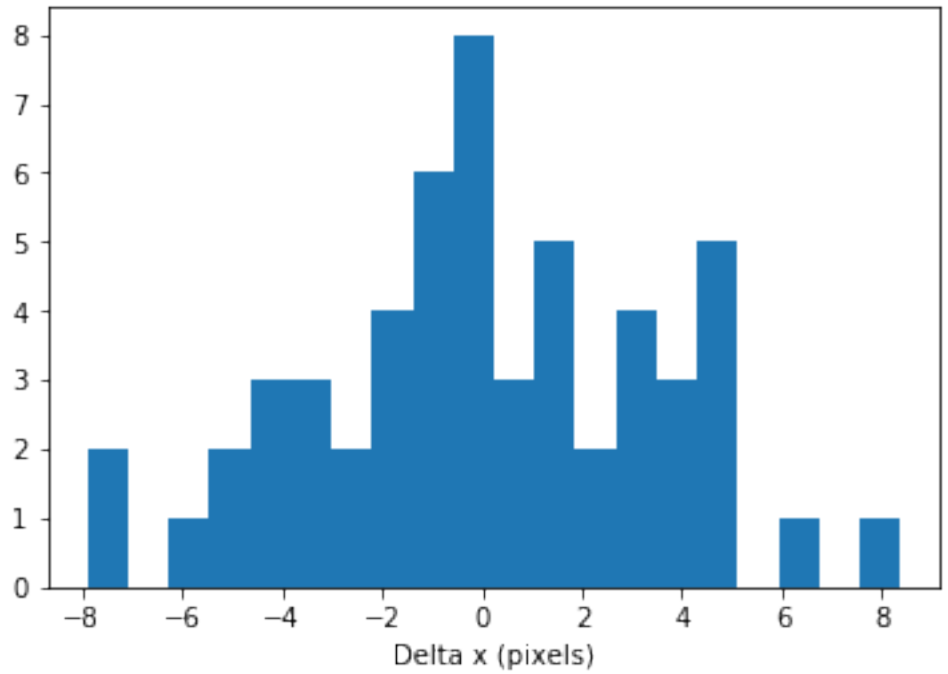
- Test this by changing Δt, and find σ^2 (with error)

- Fit plot of σ^2 vs. Δt, should show linear dependence, slope is 2D, does this (qualitatively) agree with your previous value?

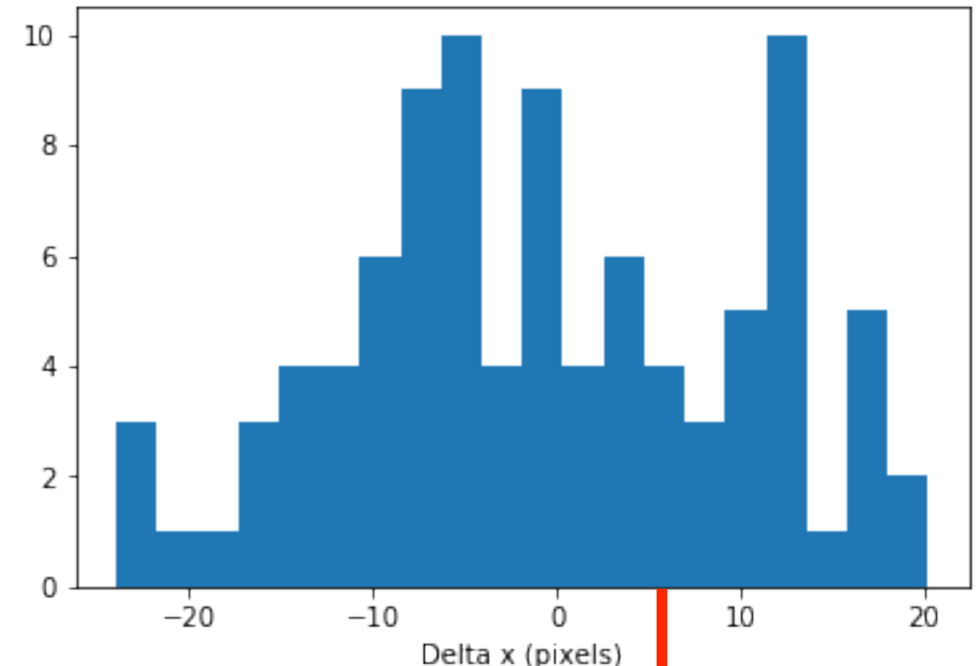
Don’t need to go crazy here, just want to see that you can fit a line and get error out

Example

Δt = 1s

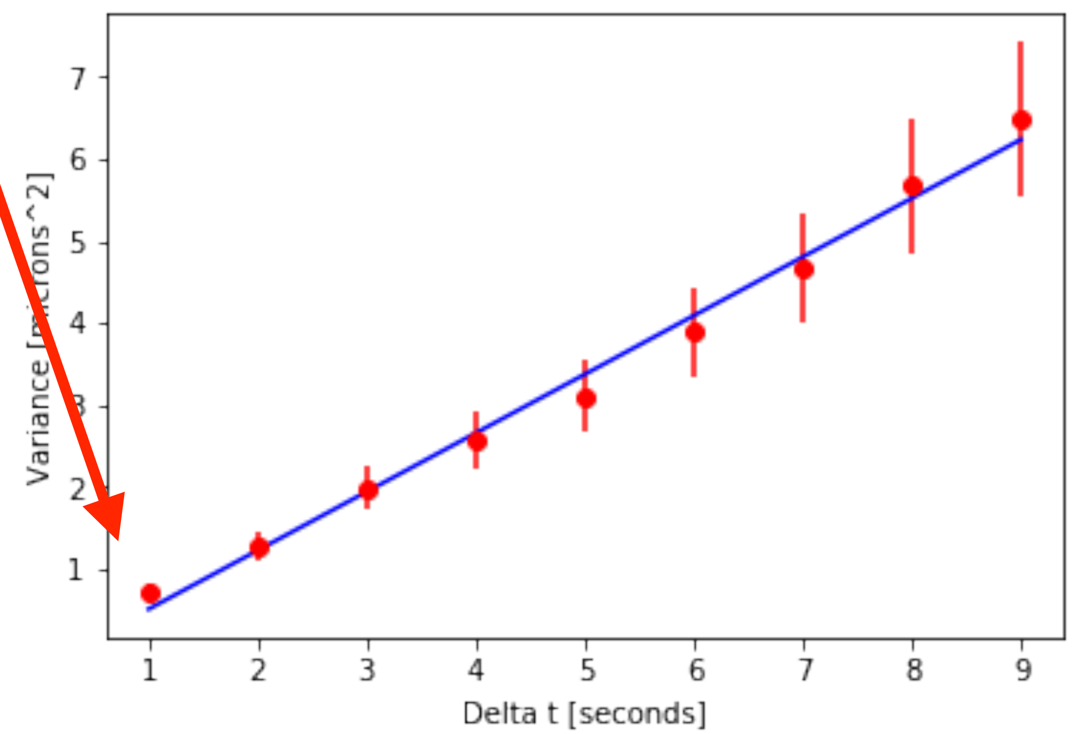


Δt = 9s



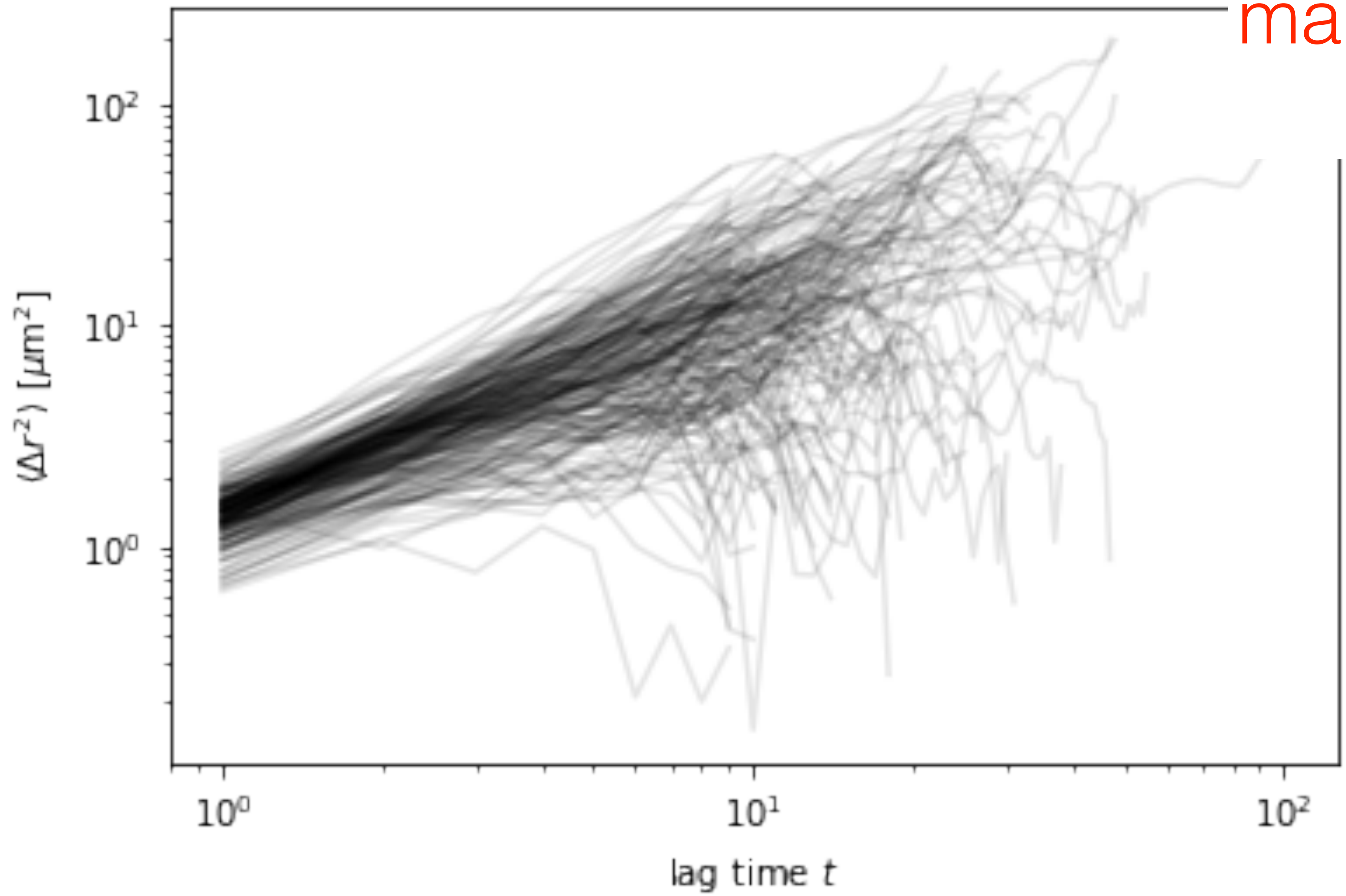
Find variance
for different Δt values

variance vs. Δt



Your Data

You will not make this exact plot…



$\Delta t \lesssim 10s$ should work

Δr^2 vs Δt for many trajectories

Any individual path may be off, but on average, there is a nice linear relationship!

Hypothesis Testing

Question

- Some ropes are tested by hanging very heavy weights from them. For good ropes, 10% fail this test.
- After being used outside for a year, you test 5 ropes and find that 2 fail (40%). Is this significant evidence that sunlight is causing the ropes to fail?
- You test 5 more ropes and 2 more fail. Does this change your conclusion about the failure rate?

Hypothesis Testing

- Frame the question as a null hypothesis
- Identify n (trials) and v (“successes”),
  for null hypothesis, identify p (expected “success” per trial)
- Check if you can use Gaussian approximation:
  Is np and n(1-p) $\gtrsim$ 10?
  If so, Gaussian w/ $\mu = np$, $\sigma^2 = np(1-p)$ works fine…
- Find statistical probability (P value) of observed v events or worse given n and p.
- Compare P value to arbitrary thresholds*

Binomial Distribution

$$B_{n,p}(\nu) = \frac{n!}{\nu!(n-\nu)!}\, p^{\nu}(1-p)^{(n-\nu)}$$

n - number of trials
p - probability of success / trial

v - number of successes

https://stattrek.com/online-calculator/binomial.aspx

Google: binomial calculator