

ACME 48X MagicMirror

What you have here is a mathematical scientific wonder, capable of creating as many as 48 copies of whatever *you* choose to place within its magic *fundamental region*.

The MagicMirror comes supplied with two special inserts.

The *cubie* can transform itself with the aid of the *MagicMirror* into a complete cube. Of course, as with all cubes, you can't see the whole thing at once, because the front side hides the stuff in the back, but you can see a lot of it. How much? Experiment and see for yourself.

The *octahedrie* can transform into an octahedron, of course. Why don't we seem to see as many copies of the octahedrie as we saw of the cubie? A good question to ponder.

One lesson to learn from the cubie and octahedrie is that the cube and the octahedron have the same kinds of symmetries—to use the jargon, they have the same *symmetry groups*. We know that must be the case, because these solids are *dual*, in the sense that if we place a dot in the center of each face of a cube the dots form the vertices of an octahedron, and if we start with an octahedron the dots will be the vertices of a cube. Any kind of symmetry either solid has will be shared by the center dot pattern and hence also by the dual solid.

The reflections in the three mirrors generate all the symmetries that these two solids have, in a precise mathematical sense. We can see in the images of the cubie and the octahedrie all the *planes of symmetry* of the solids. We can also see all of their *axes of rotational symmetry*. Just place a thin object, such as a pencil, along the joint where two mirrors meet to see some of those axes. Then switch to the other two joining lines to see the rest of the axes. Is that really all there are? Count the axes. Then count the images of the pencil. Can you see 48 images? Why not?

The cubie and octahedrie are held together with tape, which can be removed to use them as patterns for other similar inserts. If it is desired to glue such inserts permanently, a cardboard form is also provided to hold the inserts in place while the glue dries. The form illustrates another aspect of *MagicMirrors*. They come in right- and left-handed versions. To see this, simply reverse the folds on the form to observe that it now no longer fits snugly into the *MagicMirror*. But you could make a *MagicMirror* that it does fit into.

A little thought while looking at the *MagicMirror*, the cardboard form and the cubie makes it clear how the dimensions of the triangles in all three are related. The relationship between the edge lengths of the octahedrie is less obvious.

Take care in handling your *MagicMirror*, as it is made of glass. The edges have been buffed down, but they can still cut the unwary, and of course dropping the *MagicMirror* can result in 336 years of bad luck. [Do the math!] Some manufacturers wrap cloth tape around the edges, but you're old enough not to need that kind of protection.

Users sometimes ask about removing fingerprints on the *MagicMirror*. Simply treat the *MagicMirror* as you would any other piece of delicate glass, and remember that any place you can put a fingerprint you can also put a cleaning cloth.

Finally, a word about creativity. Remember that you can put anything sufficiently small into the *MagicMirror's* magical fundamental domain. Erasers, Christmas tree light bulbs, postage stamps, stuff with its own symmetry and stuff without. Stuff with designs on it or other printing. Explore! Isn't math fun!!!